SEM-IV
PHYH-C IX: ELEMENTS OF MODERN PHYSICS

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Heisenberg uncertainty principle

In classical mechanics , we can determine the position and momentum of macroscopic bodies simultaneously with perfect accuracy from its initial position and momentum and the force acting on upon it. However in quantum mechanics each moving microparticle associated with a wave packet that is extending through the region of space. Thus when a microparicle in motion its position can be any were within the wavepacket. Hence there will be a uncertainty in specifying the position of the particle. At the same time a wave packet consists of a range of wavelengths so there will be uncertainty in measurement in momentum of the microparticle. Therefore, the momentum and position of a moving microparticle cannot be measured simultaneously with perfect accuracy.

On the basis of these considerations, Werner Heisenberg in 1927enunciated principle of uncertainty
Heisenberg uncertainty principle states that the product of uncertainty in the simultaneous measurement of the position and momentum of a perticle is equal to or greater than $\hbar$,Where h is the plank constant.

i.e.

$$\Delta x \Delta p_x \geqslant \hbar \tag{0.0.1}$$

This is the position momentum Heisenberg' s uncertainty relation .

Similarly the product of uncertainty in the simultaneously measurement of the energy and time of a particle is equal to greater then $\hbar$,Where h is the plank constant

$$\Delta E \Delta t \geqslant \hbar \tag{0.0.2}$$

This is the energy -time Heisenberg 's uncertainty relation.

Example-1
Let us apply the first of the inequalities given above to study its implication in the case of hydrogen atom. Suppose the spreadΔx in the position of the electron in hydrogen atom is of the order of its radius. Now experimentally, the size of hydrogen atom is determined to be of the order of $1A^0$. Thus, uncertainty in the position of the electron must be of the order , $\Delta x \sim 10^{-10}m$, then imply the uncertainty in momentum$\Delta p \geqslant 10^{10}\hbar$. Since the mass of the electron is approximately$10^{-31}kg$, the uncertainty in the value of velocity of the electron$\geqslant 10^6 m/s$, which turns out to be much higher than the known value. It is thus evident that the electron in hydrogen atom can not be described even approximately in classical terms and it makes no sense to talk of a well defined trajectory.

Example-2

An electron remain in excited state for $10^{-11}s$. (i) What is the minimum uncertainty in the energy of an excited state? (ii)What is the physical interpretation of this uncertainty measurement of energy? (iii)Find the uncertainty in the frequency of light emitted at $10^{11}s$

Solution(i)

i) From, Heisenberg uncertainty principle.

$$\Delta E \geq \frac{\hbar}{2\Delta t}$$

Hence

$$\Delta E = \frac{h}{4\pi \cdot \Delta t}$$

$$= \frac{6\cdot 63\times 10^{-34}\ J\cdot s}{4\times 3\cdot 14\times 10^{-11}\ s}$$

$$= \frac{6\cdot 63\times 10^{-34}}{4\times 3\cdot 14\times 10^{-11}\times (1\cdot 6\times 10^{-19})}\ e\cdot v\cdot$$

$$= 3\cdot 29\times 10^{-5}\ ev.$$

(ii) The above parameter represents the limit of accuracy with which the energy of an excited state can never be measured. It gives the natural width of the excited state.

(iii) The uncertainty in frequency of radiation is

$$h\Delta\nu = \Delta E$$

$$\Delta\nu = \frac{\Delta E}{h}$$

$$= \frac{h}{(4\pi\Delta t)h}$$

$$= \frac{1}{4\times 3\cdot 14\times 10^{-11}}\ Hz = \underline{0.0796\times 10^{11}\ HZ}$$

= [illegible]

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Example-3

Assume that an electron is inside the nucleus of radius 10^-15m . Calculate from uncertainty principle the minimum kinetic energy of the electron.

(4)

Δx = maximum uncertainty in position
= diameter of the nucleus.
$= 2\times10^{-15}\,m$

using Heisenberg uncertainty principle

$$\Delta P_{min} = P_{min} = \frac{\hbar}{2\Delta x}$$

$$= \frac{1.054\times10^{-34}}{2\times(2\times10^{-15})}$$

$$= 0.263\times10^{-19}\ kgms^{-1}$$

Now

$$E_{min}^2 = P_{min}^2 C^2 + m_0^2 C^4$$

$$= (0.263\times10^{-19})^2 \times (3\times10^8)^2 + (9.11\times10^{-31})^2 \cdot (3\times10^8)^4$$

$$= (3\times10^8)^2 \times [0.069\times10^{-38} + 746.93\times10^{-46}]$$

$$E^2 = 6.2\times10^{-12}\,J$$

$$= \frac{6.2\times10^{-12}}{1.6\times10^{-19}}\ ev.$$

$$= 4.9\times10^{7}\ ev.$$

$$= 49\ MeV.$$

Non-existence of Electron and Existence of proton and neutron inside the nucleus of an atom

Date........................

* The radius r of the nucleus of an atom is in the order 10^{-14} m.

So, if the electron is considered inside the nucleus, the uncertainty in position of electron

Δx = diameter of the nucleus $\simeq 2\times10^{-14}$ m

From, uncertainty principle, the uncertainty in momentum of an electron,

$$\Delta p_x \geq \frac{\hbar}{\Delta x}$$

$$\Delta p_x \geq \frac{h}{2\pi \Delta x}$$

$$\geq \frac{6.63\times10^{-34}}{2\times3.14\times(2\times10^{-14})} \text{ kg m s}^{-1}$$

$$\geq 5.27\times10^{-21} \text{ kg m s}^{-1}$$

It mean that if the elementary particle is inside the nucleus, its minimum momentum must be

$$P_{min} = 5.27\times10^{-21} \text{ kg m s}^{-1}$$

(i) For electron: The minimum energy of an electron of mass m is obtained from relativistic formula

$$E^2 = p^2c^2 + m_0^2c^4$$
$$\simeq p^2c^2$$

Since $m_0c^2 = 0.511$ MeV is negligeble comparision to p^2c^2

$$E = pc$$
$$= 5.27\times10^{-21}\times3\times10^8 \text{ J}$$
$$\simeq 10 \text{ MeV}.$$

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So, if an electron is inside a nucleus, its energy must be of the order of 10 MeV. But from experimental data, we know that the electrons emitted by radioactive nuclei from β decay have energy only 3 to 4 MeV. Therefore, electrons can not be present within the nucleus.

② For Protons and neutrons:

For protons and neutrons, their rest mass

$$m_0 = 1.67 \times 10^{-27}\ kg$$

So, the corresponding value of energy (K.E) is given by

$$E_K = \frac{p^2}{2m_0} = \frac{(5.27 \times 10^{-21})^2}{2 \times (1.67 \times 10^{-27})}$$

$$= \frac{(5.27 \times 10^{-21})^2}{2 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19}}\ eV.$$

$$= 52\ KeV.$$

Since this energy E_K is smaller than the ~~energy~~ energies carried by these particles emitted from a nucleus, both these particles can exist inside the nucleus.

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