SEM-II
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L-5

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Fresnel Diffraction: Fresnel's Assumptions, Fresnel's Half-Period Zones for Plane Wave

Fresnel's Assumptions:

In order to explain diffraction phenomenon, Fresnel made the following assumptions.

• The entire wavefront can be divided into a large number of elements or zones of small area such that each of these elements acts as a source of secondary waves emitting waves in all directions.

• The effect at any point "O" will be the resultant of the secondary wavelets reaching"O" from various elements of the wavefront.

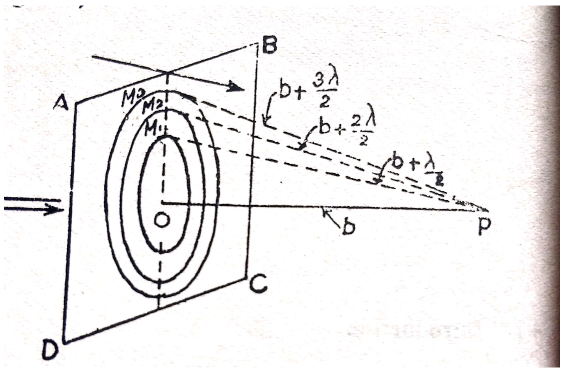
• The resultant effect at any point will be combination of the effect of all secondary waves reaching at that point.

• The effect at any point due to a particular zone depends on (a). the distance of point from ilie zone. (b). the inclination of the point with reference to zone under consideration.(c). area of the zone.

Fresnel treatment of the plane wavefront: Fresnel half period zone.

Fresnel explained diffraction on the basis of mutual interference of the secondary wavelets from various point of the wavefront. The wavefront may be plane , spherical or cylindrical depending on the source.

For simplicity let us take the case of plane wavefront $ABCD$ traveling towards the right and calculate its effects on an external point P.



Divide the whole wavefront into a number of zones called Fresnel's half period zones in the following manner. From Pdraw a normal PO to the surface $ABCD$. Let $PO = b$.

The foot of the normal O is called the pole of the wavefront. With P as centre and radii sphere are drawn as

$$(b + \frac{\lambda}{2}), \ (b + \frac{2\lambda}{2}), \quad (b + \frac{3\lambda}{2}) \quad .,.,.,$$

The section of the spheres by the plane wavefront $ABCD$ will be circles $M_1, \ M_2 \ \ M_3$etc. So the circumference of the first circle will be at a distance $(b+\frac{\lambda}{2})$, the second circle at $(b+\frac{2\lambda}{2})$ and third $(b+\frac{3\lambda}{2})$ and so on from P. The area enclosed by the first circle is called the first half period zone, the annular area between the second and the first circle is called the second half period zone and so on.

The area of the first half period zone $\quad = \pi(OM)^2$
$= \pi\{(b + \frac{\lambda}{2})^2 - b^2\} = \pi b\lambda$
Similarly the area of the second half period zone
$= \pi(OM_2)^2 - \pi(OM_1)^2 = \pi[\{(b+\frac{2\lambda}{2})^2 - b^2\} - \{(b+\frac{\lambda}{2})^2 - b^2\}]$
$= \pi(2b\lambda - b\lambda) = \pi b\lambda$
Similarly the area of the nth half period zone
$= \pi(OM_n)^2 - \pi(OM_{n-1})^2 = \pi[\{(b + \frac{n\lambda}{2})^2 - b^2\} - \{(b + (n - 1)\frac{\lambda}{2})^2 - b^2\}]$
$= \pi b\lambda + \pi(2n - 1\frac{\lambda^2}{4}) \equiv \pi b\lambda$

Thus we see that the area of each half period zone is approximately equal.

The radius of the nth circle(neglecting the terms involving λ^2.) .i.e.
$= OM_n = \{(b + \frac{n\lambda}{2})^2 - b^2\}^{\frac{1}{2}} = \sqrt{nb\lambda}$
The radious of the first zone $OM_1 = \sqrt{b\lambda}$, that of the second $OM_2 = \sqrt{2b\lambda}$,.... etc. Thus the outer radii of the zones are

proportional to the square root of their zone number.

Now coming back to Huygen's principle every point onthe wavefront may be regarded as a source of secondary wavelets. But the secondary wavelets from different points, though they start with the same phase $\phi$from $ABCD$ will reach P with a phase difference. Since the successive zones differ by an average path difference $\frac{\lambda}{2}$ with respect to p, the wave disturbances from the zones will differ by a phase difference of π

Let $d_1, d_2, d_3, ...$ be the amplitude at P due to the wavelets coming from $1st, 2nd, 3rd$ half period zones respectively. The resultant amplitude at P will be

$$D = d_1 - d_2 + d_3 - d_4.... + (-1)^n d_n \qquad (1)$$

In order to compute D following factor s are to be considered

- The zone area increases slightly with the zone number.
- The inverse variation of amplitude with the average distance of the zone.
- The decrease of amplitude with obliquity of the zone.$d = (1 + cos\theta)$

Now taking $nodd$ in equation(1)

$$D = \frac{d_1}{2} + [\frac{d_1}{2} - d_2 + \frac{d_2}{2}] + [\frac{d_3}{2} - d_4 + \frac{d_5}{2}] + \frac{d_n}{2} \qquad (2)$$

for n even

$$D = \frac{d_1}{2} + [\frac{d_1}{2} - d_2 + \frac{d_3}{2}] + [\frac{d_1}{2} - d_4 + \frac{d_5}{2}] + \frac{d_{n-1}}{2} - d_n \quad (3)$$

But $\frac{d_1+d_3}{2} = d_2, \frac{d_3+d_5}{2} = d_4$ and so on.

We can take the contribution of successive zones as approximately equal and opposite. Beside if $n$becomes very large

$d_n or d_{n-1}$ can be neglected due to obliquity factor.
So

$$D = \frac{d_1}{2} \tag{4}$$

or the amplitude at P due to the whole wavefront is equivalent to half of the contribution from the first half period zone.