SEM-IV
PHYH-C IX: ELEMENTS OF MODERN PHYSICS

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wave function,Probability and probability density, physical interpretation of a wave function

wave function

So in previous lecture we show that a wave is associated with the moving quantum mechanical particles

The space time behavior of each moving quantum mechanical particle can be describe by a function. The function is known as wavefunction. It is generally denoted as $\psi(\bar{\mathbf{x}}, \mathbf{t})$. This measure the probability of finding a particle about a position.

The magnitude of $|\psi(\bar{\mathbf{x}}, \mathbf{t})|$ is large in the region where the probability of finding the particle is high and is small in the regions where the probability of finding is low. Hence , the wavefunction $|\psi(\bar{\mathbf{x}}, \mathbf{t})|$ measure the probability of the particle around a particular position.

Probability and probability density:

We know the wavefunction $|\psi(\bar{\mathbf{x}}, \mathbf{t})|$ of a quantum mechanical gives the space time behaviour of a particle.

The probability density of a particle is the probability of finding the particle per unit volume of a given space at a particular time. It is generally expressed as the product of normalized wavefunction and its complex conjugate ψ^*.

So probability density

$$P = \psi(\bar{\mathbf{r}}, \mathbf{t})\psi(\bar{\mathbf{r}}, \mathbf{t})^* \tag{0.0.1}$$
$$= |\psi(\bar{\mathbf{r}}, \mathbf{t})|^2 \tag{0.0.2}$$

So the probability that the particle will be found in the volume element $d\tau = \psi^*\psi d\tau$.

Since the particle must be somewhere in the space the total probability to find the particle in space should be equal to 1.

$$P = \int_{-\infty}^{\infty} \psi(\bar{\mathbf{r}}, \mathbf{t})\psi(\bar{\mathbf{r}}, \mathbf{t})^* d\tau = 1 \tag{0.0.3}$$

In one dimension case the probability of locating the particle in a distance dx is given by

$$Pdx = \psi^*\psi dx \tag{0.0.4}$$

Since the particle must be somewhere along X axis the total probability for one dimensional motion over all values of x must be 1.

$$\int_{-\infty}^{\infty} \psi^*\psi dx = 1 \tag{0.0.5}$$

physical interpretation of a wave function

The important significance of wavefunction are:

• It gives the space-time behaviour of each quantum mechanical particle .

• it measure the probability of finding a particle about a position.

• Its magnitude is large in the regions where the probability of finding the particle is high and is small in a region where the probability of finding the particle is low.

• It should be continuous ,single valued and finite.

Limitation of wave function

• The wavefunction $\psi$and its space derivative $\frac{\delta\psi}{\delta x}, \frac{\delta\psi}{\delta y}, \frac{\delta\psi}{\delta z}$ must be continuous ,single valued and finite everywhere.

• The wave function ψ must be continuous in all region except where the region potential energy $V(x, y, z) = \infty$.

Normalization of wave function

• If $\psi(x, t)$ is multiplied by a complex constant N such that $\psi_N(x, t) = N\psi(x, t)$ Where $\psi_N(x, t)$ satisfy the relation

$$\int_{-\infty}^{\infty} |\psi_N(x, t)|^2 dx = |N|^2 \int_{-\infty}^{\infty} |\psi(x, t)|^2 dx = 1 \tag{0.0.6}$$

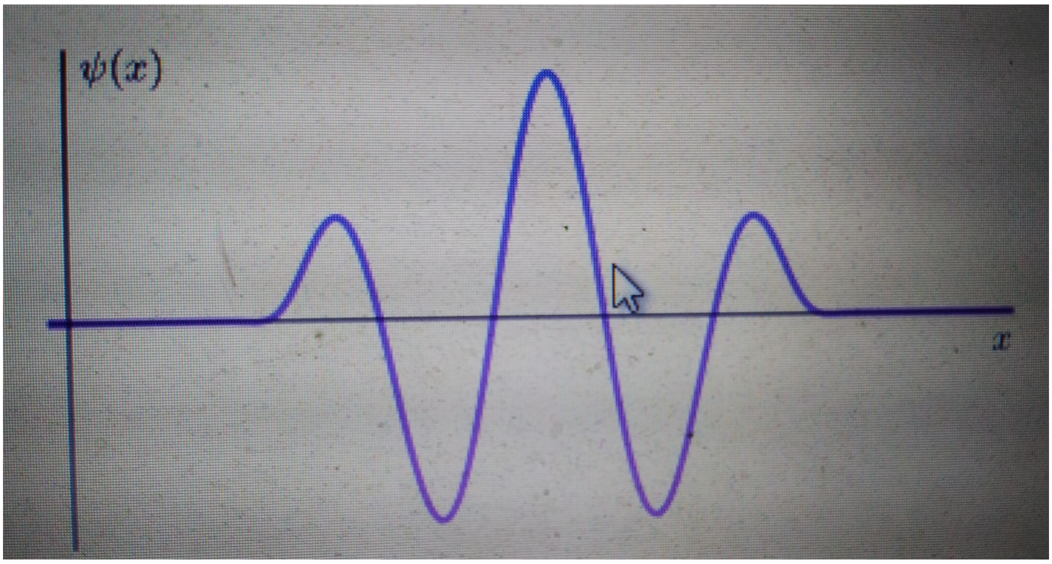
Then$\psi_N(x, t)$ is said to be normalized wave function.

Then

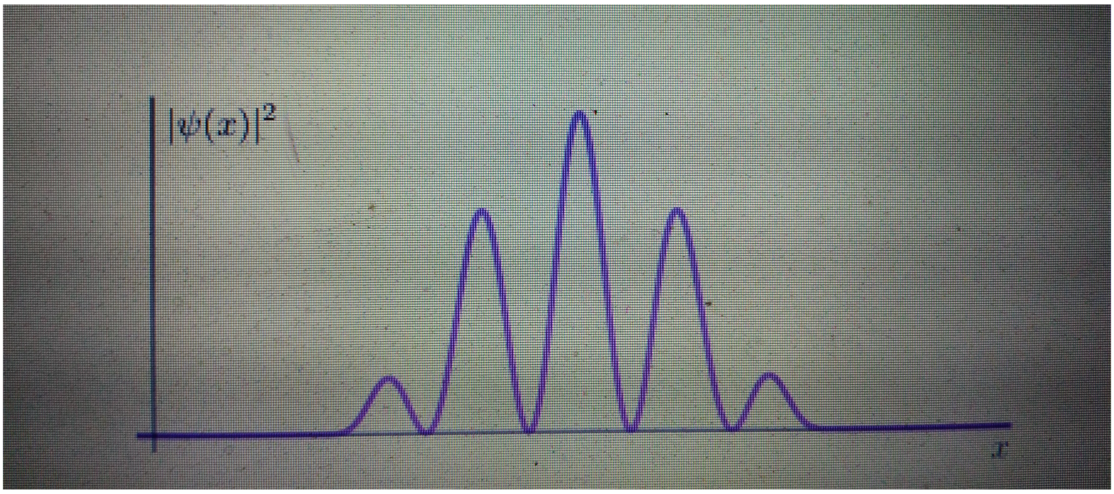
$$|N|^2 = \frac{1}{\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx} \tag{0.0.7}$$

Is called the normalization constant. Obviously a wave function is normalizable if $\int_{-\infty}^{\infty} |\psi(x, t)|^2 dx$ or more generally $\int_{\tau} |\psi(x, t)|^2 d\tau$ over all space remain finite. This is known as square-integrability of the wave function.

Example
For example, if we draw the x axis across the two-slit interference pattern. Then the wavefunction of each particle, just before it hits the detection screen, might look something like this:



This wavefunction has five "bumps," corresponding to the five bright lines in the interference pattern. The dark lines in the pattern are at the locations where the wavefunction is zero. More precisely, the brightness of the interference pattern is proportional to the square of the wavefunction, in analogy to the way the energy in a mechanical wave or an electromagnetic wave is proportional to the square of the wave amplitude. Here is a plot of the square of our five-bump wavefunction:



The height of this graph at any x is then proportional to the probability of finding the particle at$x$, when the subsequent interaction with the detection screen "measures" the particle's position. After many such measurements are made on identically prepared particles, the five-line interference pattern emerges.