2015

Subject : Physics (Honours)

Paper : II

Time: 2 Hours **Full Marks: 50**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

New Syllabus

Group – A

SECTION–I

Answer *any five* of the following questions: 2×5=10

1. (a) If a particle moves in a path with constant speed, show that the acceleration is either zero or perpendicular to the velocity.

(b) A particle of mass m moves in a linear potential $v = Kr$. For what energy and angular momentum, will the orbit be a circle of radius R about the origin?

(c) Two satellites A and B of the same mass are orbiting the earth at altitudes R and $3R$ respectively, where R is the radius of the earth. Taking their orbits to be circular obtain the ratio of their Kinetic energies.

(d) Define conservative force. We can always associate a potential with a conservative force—Explain.

(e) Prove that the motion of a particle subject to a central force is confined to a plane.

(f) Find the equation of motion of a rocket moving vertically in a uniform gravitational field.

(g) Two circular metal discs have the same mass M and same thickness t. Disc 1 has a uniform density ρ_1 which is less than the uniform density of disc 2 i.e. ρ_2. Which disc has larger moment of inertia?

(h) Prove that the torque acting on a particle equals to the time rate of change of its angular momentum.

SECTION–II

Answer *any one* of the following questions: 12×1=12

2. (a) What do you mean by elastic and inelastic collisions? 2

(b) A neutron of mass m undergoes an elastic head on collision with a nucleus of mass M, initially at rest. By what fraction in the kinetic energy of the neutron reduced? 5

(c) Given that the moment of inertia of a cube about an axis passing through the centre of mass and centre of one face is I_0, find the moment of inertia about an axis passing through the centre of mass and one corner of the cube. 5

3. (a) Three point masses m_1, m_2 and m_3 interact with each other through gravitational force. Write down the equations of motion of the particles. Determine the angular frequency of rotation of the system so that the distance between any two particles always remains equal to 'd'. 4

(b) Show by means of substitution $r = \frac{1}{u}$ that, the differential equation for the path of the particle in a central force field in given by,

$$\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}$$

Where $r^2\theta = h$ and the other symbols have their usual meaning. Using the above equation show that, if the law of central force is defined by $f(r) = -K/r^2; K > 0,$ then the path of the particle is a conic. 4+4

SECTION III

Answer *any two* of the following questions: 6×2=12

4. Two particles having masses m_1 and m_2 move in such a way that, their relative velocity $\vec{v}$ and the velocity of their centre of mass is $\vec{V}$. Prove that, the total kinetic energy is $\frac{1}{2}\mu v^2 + \frac{1}{2}MV^2$; where μ and M are the reduced mass and the total mass respectively. 6

5. A long range rocket is fired from the surface of the earth (radius R) with a velocity (v_r, v_θ). Obtain an equation of motion to determine the maximum height (H) attained by the rocket. Show that in the lowest order of $\left(\frac{H}{R}\right)$, $\vec{v}$ is vertical. 3+2+1

6. (a) State and prove the perpendicular axes theorem.

(b) Use it to determine the moment of inertia of a uniform circular disc about one of its diameter. 2+4

GROUP B

SECTION I

7. Answer *any two* from the following questions: 2×2=4

(a) The tension of a steel wire of radius 0.5 mm stretched between two rigid supports $1m$ apart is found to be $37.7N$ when the temperature is 20°C. At what temperature will the tension on the wire just vanish? Assume Young's modulus for steel= $2 \times 10^{11} N/m^2$ and co-efficient of linear expansion of steel in 1.2×10^{-5}/°C .

(b) If the mass of a gas inside a soap bubble is doubled, find the relation connecting the new radius (r_2) and the old one (r_1).

(c) Compare loads required to produce equal deflections for two beams made up of same material and having same length and weight. One beam has circular and the other has square cross section.

(d) A soap bubble of radius r_1 is blown at the end of a capillary tube of length $'l'$ and internal radius $'a'$. Find the time taken by the bubble to be reduced to a radius r_2, given the viscosity of air at the temperature as η.

Section II

Answer *any two* from the following questions: 6×2=12

8. (a) Explain in your own words, what do you mean by streamline and turbulent motion of a liquid? What is Reynold's number? 2+1

(b) Liquid flows in a streamline motion through a horizontal tube of length l and radius $'a'$under an effective pressure P. If a cylindrical co-axial rod of radius $^{a}/_{\sqrt{2}}$ and length $'l'$ is inserted into the tube, calculate the percentage of reduction of the rate of flow for the same pressure difference between the ends. 3

9. Establish the relations connecting Young's modulus, bulk modulus, rigidity modulus and Poisson's ratio of a maerial. 6

10. Explain with suitable diagram, the principle of operation of a Langmuir's type of diffusion pump. 6

Old Syllabus

Time: 2 Hours **Full Marks: 50**

Group A

1. Answer *any four* from the following questions: 2×4=8

(a) Obtain the relation between surface tension and surface energy.

(b) What will be the change in entropy when one mole of an ideal gas is allowed to expand freely to double its original volume?

(c) With what velocity will an air-bubble with a diameter of $2mm$, rise in water?

(Given $\eta = 0.01$ poise).

(d) Show that the strain energy of a twisted wire in $\frac{1}{2}.C_m.\theta_m$; where C_m is the couple for maximum twist θ_m.

(e) What do you mean by thermometric conductivity? How does it differ from thermal canductivity?

(f) The efficiency of a carnot engine can be increased by either increasing the temperature of the source or by reducing temperature of the sink. Which arrangement is more advantageous in your opinion?

(g) Calculate the mean free path of molecules of H_2 gas at 20℃ at atmospheric pressure. Assume the molecular diameter to be $2 \times 10^{-10}m$.

GROUP B

Answer *any two* from the following: 12×2=24

2. (a) Caculate the gravitational potential for a thin spherical shell at a point on its surface. Explain the cases when the point in (i) outside the shell and (ii) inside the shell. 4+1+1

(b) The tube of a mercury barometer is 3.0 mm in diameter. What error is introduced into the reading because of surface tension(S)? For mercury $S = 465$ dynes per cm, angle of contact = 128°, density of mercury= 13.6 gm/cm^3, $g = 980\ cm/sec^2$ 3

(c) Explain the terms: neutral surface, neutral axis and axis of bending in connection with bending of beams. 3

3. (a) Water flows through a horizontal tube of length 0.2 m and internal radius 8.1×10^{-4} m under a constant head of liquid 0.2m high. In 12 mins, $864 c.c$ of liquid comes out from the tube. Calculate the viscocity coefficient of water. Also verify that the conditions of streamline flow exist. 3

(b) A tank consists CO_2 at a pressure of 6 atmosphere and 40°C. A leak occurs in the tank which is not detected until the pressure falls to 4 atmosphere and temperature at that time was 22°C . Find the mass of CO_2 that has leaked out if the orignal mass was 30 kg. 3

(c) Using Maxwell's velocity distribution law, derive the expressions for average velocity, r.m.s velocity and most probable velocity of the gas molecules. 6

4. (a) Let a cylinder has an electrically heated wire along its axis. Discuss the heat flow in radial direction and find an expression for the temperature distribution. How can you estimate the conductivity of the material of the cylinder? 8+2

(b) Write down the Maxwell's thermodynamical relations. 2

5. (a) What do you mean by λ-transition? What are the distinguishing features between 1st order and λ-transition? 2+2

(b) Show that specific heat of steam is negative. 4

(c) Sketch and explain the Carnot cycle for an ideal gas on a ; (i) $T - S$ diagram; (ii) $P - T$ diagram, (iii) $v - T$ diagram and (iv) $u - v$ diagram. 4

GROUP C

Answer *any three* taking atleast one from each section. 6×3=18

SECTION I

6. Establish the relations connecting Young's modulus, bulk modulus, rigidity modulus and Poisson's ratio of a material. 6

7. (a) If the mass of a gas inside a soap bubble is doubled, find the relation connecting the new radius (r_2) with the old one (r_1). 2

(b) A thin wire of radius b is ploced co-axially in a narrow tube of length l and radius a . Find the volume of the liquid flowing through the annular space between the wire and the tube when a pressure difference P is maintained between the ends of the tube. 4

SECTION II

8. (a) Prove that entropy is a state function.

(b) Calculate the change in entropy when $m\ gm$ of ice at $T_1 K$ be converted into steam at $T_2 K$. 3+3

9. (a) Show that the ratio of adiabatic and isothermal bulk modulus of elasticity= γ . 4

(b) The specific heat value of water at 0°C increases by 9.1% on freezing and latent heat of ice is 80 cal/gm at atmospheric pressure. Calculate the pressure needed to lower the melting point of ice by 1°C. 2

10. (a) Prove that coefficient of viscocity of a gas is independent of its density.

(b) A pond is covered with ice, 0.04m thick. The temperature of the air above is 261K. At what rate will the ice thicken?
[Given k of ice = 2.184 w/m/K density=920kg/m^3 and latent heat=33kJ/kg] 3+3

B.Sc PHYSICS (Hons) Examination – 2015

PAPER – II

(New Syllabus)

Model answers

Group – A

SECTION – I

Answer any five of the following questions: $2 \times 5 = 10$

1. (a) [Here speed is constant and not the velocity.]

If the velocity is $\vec{v}$ and corresponding magnitude or speed is v, then, $\vec{V}\cdot\vec{V} = v^2$.

So, $\frac{d}{dt}(\vec{V}\cdot\vec{V}) = \frac{d}{dt}(v^2) = 2v\frac{dv}{dt} = 0$ [As v is constant].

So, $\vec{V}\cdot\frac{d\vec{V}}{dt} = 0$

Hence $\frac{d\vec{V}}{dt}$ is either 0 or perpendicular to $\vec{V}$.

(b) The force acting on the particle $\vec{F} = -\frac{dv}{dr}\hat{r} = K\hat{r}$.

If the particle moves in a circular orbit of radius R, then we may write,

$$m\omega^2 R = K$$

$$\therefore \omega^2 = \frac{K}{mR}.$$

$\therefore$ The energy of the particle is $E = Kr + \frac{1}{2}mv^2 = Kr + \frac{1}{2}m\omega^2R^2$
$= \frac{3KR}{2}$ (1 mark)

Angular momentum about the origin is,

$$L = m\omega R^2 = mR^2\sqrt{\frac{K}{mR}} = \sqrt{mKR^3}$$ (+1 mark).

(c) The velocity of an artificial satellite having mass M and at an altitude h from the earth's surface can be given by,

$$v = \sqrt{\frac{GM}{R+h}}$$ where R = Radius of earth

So velocity of satellite A, $v_A = \sqrt{\frac{GM}{2R}}$

" " Satellite B, $v_B = \sqrt{\frac{GM}{4R}}$.

Hence ratio of kinetic energy $\frac{E_A}{E_B} = \frac{\frac{1}{2}Mv_A^2}{\frac{1}{2}Mv_B^2} = \frac{GM}{2R}\cdot\frac{4R}{GM}$.

$= 2:1$

(d) The forces acting on a body are said to be conservative if the work done by the forces in causing displacement of the body depends only on the initial and final positions of the body and not on the path traversed from the initial to the final point. Thus for an arbitrary closed path, the work done by a conservative force $\vec{F}$ will be zero. This may be represented as;

$$\oint \vec{F}.\vec{dr} = 0. \qquad [1]$$

Using Stoke's theorem; the above equation may be written as,

$$\vec{\nabla} \times \vec{F} = 0.$$

As curl of a gradient always vanishes, $\vec{F}$ must be gradient of a scalar. Hence,

$$\vec{F} = -\vec{\nabla} V.$$

Here V is called potential energy. Thus we can always associate a potential with a conservative force. [1]

(e) A central force may be expressed as,

$$\vec{F}(r) = f(r)\,\hat{r}$$ where $\hat{r}$ is the unit vector in the direction of $\vec{r}$.

Hence $\vec{r} \times \vec{F} = 0$.

Again $\vec{F} = m\frac{d\vec{v}}{dt}$; so, $\vec{r} \times \frac{d\vec{v}}{dt} = 0$

or $\frac{d}{dt}(\vec{r} \times \vec{v}) = 0$

or $\vec{r} \times \vec{v} = \vec{h}$, a constant vector.

$\therefore\ \vec{r}.\vec{h} = \vec{r}.(\vec{r} \times \vec{v}) = (\vec{r} \times \vec{r}).\vec{v} = 0.$

Hence $\vec{r}$ is perpendicular to the constant vector $\vec{h}$ and thus the motion takes place in a plane.

(f) Consider a rocket, having mass M and velocity v, in the time interval Δt, ejecting a mass ΔM at a velocity v_0 relative to the rocket and giving an additional velocity Δv. Now, taking the vertical upward direction as positive, we have

$$(M-\Delta M)(v+\Delta v) + (v+u_0)\Delta M - Mv = -Mg\,\Delta t$$

or $$M\frac{\Delta v}{\Delta t} = -v_0\frac{\Delta M}{\Delta t} - Mg$$

Taking the limit, $\Delta t \to 0$,

$$M\frac{dv}{dt} = -v_0\frac{dM}{dt} - Mg.$$

This is the required equation of motion.

(g) Let the radii of the discs be r_1 and r_2 respectively.
As the discs have same mass and thickness, we have $\rho_1 r_1^2 = \rho_2 r_2^2$.

$$\therefore \frac{r_1^2}{r_2^2} = \frac{\rho_2}{\rho_1}.$$

The moment of inertia may be given as, $I_1 = \frac{M r_1^2}{2}$ and $I_2 = \frac{M r_2^2}{2}$.

$$\therefore \frac{I_1}{I_2} = \frac{r_1^2}{r_2^2} = \frac{\rho_2}{\rho_1}.$$

Since $\rho_1 < \rho_2$ as given, hence $I_1 > I_2$.
$\therefore$ The disc 1 has the larger moment of inertia.

(h) We can define torque as, $\vec{r} \times \vec{F} = \vec{r} \times \frac{d}{dt}(m\vec{v})$
where the symbols have usual meaning.

$\therefore$ Angular momentum $\vec{L} = \vec{r} \times \vec{p}$ ($\vec{p}$ = linear momentum).

$$= \vec{r} \times m\vec{v}.$$

$$\therefore \frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{v}) = \frac{d\vec{r}}{dt} \times m\vec{v} + \vec{r} \times \frac{d}{dt}(m\vec{v})$$

$$= \vec{v} \times m\vec{v} + \vec{r} \times \vec{F}.$$

$$= 0 + \vec{r} \times \vec{F} = \vec{r} \times \vec{F} = \text{torque}.$$

Thus the torque acting a particles equals to the time rate of change of its angular momentum.

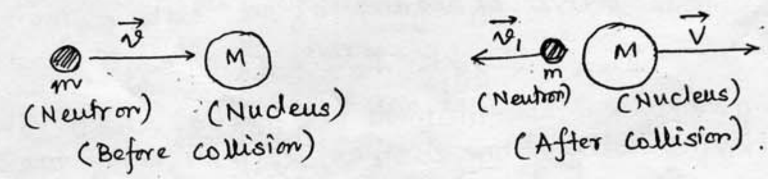
SECTION — II

Answer any <u>one</u> of the following questions :- $12 \times 1 = 12$.

(2) (a) If the forces of interaction between the colliding particles are conservative, the kinetic energy remains conserved in the collision process and the collision is said to be elastic. [1]

When the kinetic energy is changed in the collision process, but the momentum as well as total energy remains consered, then it is called inelastic collision. [1].

(b)



Let us take the neutron of mass m collides with a nucleus of mass M at rest with a velocity $\vec{v}$. After collision, the neutron retraces with a velocity $\vec{v}_1$ and the nucleus recoils with a velocity $\vec{V}$.

Now using the law of conservation of energy before and after collision, we can write,

$$\frac{1}{2}mv^2 = \frac{1}{2}m v_1^2 + \frac{1}{2}MV^2 \quad \text{———— (i)}$$

Using conservation of linear momentum we can write,

$$mv = mv_1 + MV \quad \text{———— (ii)}$$

[1]

Taking square of both sides of the above equation,

$$m^2v^2 = m^2v_1^2 + M^2V^2 + 2mMv_1V$$

$$\text{or} \quad \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\frac{M^2}{m}V^2 + Mv_1V \quad \text{———— (iii)}$$

Now, comparing equation (i) & (iii) we write, [1]

$$\frac{1}{2}MV^2 = \frac{1}{2}\frac{M^2}{m}V^2 + Mv_1V$$

$$\text{or} \quad V^2 = \frac{M}{m}V^2 + 2v_1V$$

$$\text{or} \quad \left(1 - \frac{M}{m}\right)V^2 = 2v_1V$$

$$\text{So,} \quad V = \frac{2}{\left(1-\frac{M}{m}\right)}v_1 \quad \text{———— (iv)}$$

[1]

Substituting eq$\underline{n}$ (iv) in eq$\underline{n}$ (ii), we get,

$$mv = mv_1 + \frac{2M}{\left(1-\frac{M}{m}\right)}v_1 = mv_1\left(1 + \frac{2M}{m-M}\right)$$

$$= mv_1\,\frac{m+M}{m-M}$$

$$\text{i.e} \quad v_1 = -\frac{M-m}{M+m}v \quad \text{———— (v)}$$

[1]

$\therefore$ The required fraction of kinetic energy loss is,

$$\frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{M-m}{M+m}\right)^2v^2}{\frac{1}{2}mv^2}$$

$$= 1 - \left(\frac{M-m}{M+m}\right)^2 = \frac{4mM}{(M+m)^2} \quad \text{———— (vi)}$$

[1]

(c)

If we use cartesian coordinates with origin at the centre of mass and the axes through the centres of the three pairs of faces of the cube; then we can have, $I_{xx} = I_{yy} = I_{zz} = I_0$ (say).

In general, the moment of inertia about an axis having direction cosines α, β, γ, may be given as,

$$I = \alpha^2 I_{xx} + \beta^2 I_{yy} + \gamma^2 I_{zz} - 2\alpha\beta I_{xy} - 2\beta\gamma I_{yz} - 2\gamma\alpha I_{zx}$$

$= (\alpha^2+\beta^2+\gamma^2)\, I_0 .$ [2]

To find the direction cosines of a radius vector $\vec{r}$ from the origin to one corner of the cube,

$$\vec{r} = a\hat{i} + a\hat{j} + a\hat{k} .$$

Where we have taken length of a side of the cube as 2a.

$$\therefore\ |\vec{r}| = \sqrt{3}\, a .$$

$$\therefore\ \alpha = \beta = \gamma = \frac{1}{\sqrt{3}}$$ [2]

Putting we get, $I = I_0 .$ [1]

(3)

(a) The equation of motion of ith particle may be given as,

$$m_i \frac{d^2\vec{r}_i}{dt^2} = -\sum_{j\neq i}^{3} \frac{G\, m_i m_j}{r_{ij}}\ \vec{r_{ij}} \quad \text{———— (i)}$$ [1]

Where $\vec{r_{ij}} = \vec{r_i} - \vec{r_j}$ (taken) and $-$ sign indicates the force of attraction.

$$\text{So, } \ddot{\vec{r_i}} = \frac{G}{d^3} \sum_{j\neq i}^{3} m_j (\vec{r_j} - \vec{r_i})$$

$$= \frac{G}{d^3}\left[-\sum_{j\neq i}^{3} m_j \vec{r_i} + \sum_{j\neq i}^{3} m_j \vec{r_j}\right]$$

$$= \frac{G}{d^3}\left[-\sum_{j\neq i}^{3} m_j \vec{r_i} - m_i \vec{r_i} + \sum_{j} m_j \vec{r_j}\right]$$

$$= \frac{G}{d^3}\left[-\vec{r_i} \sum_{j=1}^{3} m_j + M\, \vec{R}_{CM}\right]$$ [According to the def^n of CM]

$$= -\frac{GM}{d^3}\vec{r_i} \quad \text{———— (ii)}$$ [Choosing $R_{CM} = 0$]

Choice of CM as the origin of the system, each mass is attracted towards the C.M. of the system and the force is harmonic. [2]

So with d remaining constant, the system will rotate with an angular frequency ω given by,

$$\omega = \sqrt{\frac{GM}{d^3}} .$$ [1]

③ (b) We have given $r^2\dot{\theta} = h$ and $r = \frac{1}{u}$.

Hence, $\dot{\theta} = \frac{h}{r^2} = hu^2$.

So from equation of motion,

$$m(\ddot{r} - r\dot{\theta}^2) = f(r)$$

or $m\left(\ddot{r} - \frac{h^2}{r^3}\right) = f(r)$. [1]

Now $\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = -h\frac{du}{d\theta}$

$\therefore \ddot{r} = -h^2u^2\frac{d^2u}{d\theta^2}$ [1]

So putting this value in the above eqn we get

$$m\left(-h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3\right) = f\left(\frac{1}{u}\right)$$

or $\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}$ [2]

Again, $f(r) = -\frac{K}{r^2}$ where $K > 0$.

$\therefore f\left(\frac{1}{u}\right) = -Ku^2$.

$\therefore \frac{d^2u}{d\theta^2} + u = \frac{K}{mh^2}$ [1]

Solving we get,

$$u = A\cos\theta + B\sin\theta + \frac{K}{mh^2}$$

or $u = \frac{K}{mh^2} + C\cos(\theta - \phi)$

or $r = \frac{1}{\frac{K}{mh^2} + C\cos(\theta - \phi)}$ [1]

It is always possible to choose the axes so that $\phi = 0$;

Hence $r = \frac{1}{\frac{K}{mh^2} + C\cos\theta} \rightarrow$ This is a form of conic.

Again, the general form of a conic can be given by,

$$r = \frac{p}{1 + \epsilon\cos\theta} = \frac{1}{\frac{1}{p} + \frac{\epsilon}{p}\cos\theta}$$ [1]

Comparing, we may write,

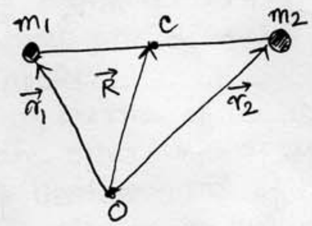
$$\frac{1}{p} = \frac{K}{mh^2} \quad \text{or} \quad p = \frac{mh^2}{K}$$

$$\text{and } \frac{\epsilon}{p} = c \longrightarrow \epsilon = \frac{mh^2c}{K} \qquad [1]$$

SECTION — III

Answer any two questions. $6 \times 2 = 12$

(4) Let $\vec{r_1}, \vec{r_2}$ and $\vec{R}$ be the position vectors with respect to origin (O) of mass m_1, mass m_2 and centre of mass C respectively.



From the definition of the centre of mass, we have,

$$\vec{R} = \frac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1+m_2} \qquad [1]$$

$$\vec{V} = \frac{m_1\vec{v_1} + m_2\vec{v_2}}{m_1+m_2} \qquad [1]$$

Again $m_1\vec{v_1} + m_2\vec{v_2} = (m_1+m_2)\vec{V}$

and $\vec{v} = \vec{v_1} - \vec{v_2}$

$$\therefore \vec{v_1} = \vec{V} + \frac{m_2\vec{v}}{m_1+m_2} \quad \text{and} \quad \vec{v_2} = \vec{V} - \frac{m_1\vec{v}}{m_1+m_2} \qquad [1+1]$$

$$\text{Total kinetic energy} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}m_1\left(\vec{V} + \frac{m_2\vec{v}}{m_1+m_2}\right)^2 + \frac{1}{2}m_2\left(\vec{V} - \frac{m_1\vec{v}}{m_1+m_2}\right)^2$$

$$= \frac{1}{2}MV^2 + \frac{1}{2}\mu v^2$$

Where $M = m_1+m_2$ = total mass and $\mu = \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass of the system. [2]

(5) Under the action of central force field, both angular momentum and total energy is conserved, so we may write,

$$mRv_\theta = m(R+H)v_\theta \text{ ——— (i)}$$

$$\frac{1}{2}m\left(v_r^2 + v_\theta^2\right) - \frac{GmM}{R} = \frac{1}{2}mv_\theta'^2 - \frac{GmM}{R+H} \text{ ——— (ii)}$$

Here dash (') corresponds to the final stage of the rocket and so $v_r' = 0$.

Where m and M being the masses of rocket and earth.
So from (i) and (ii) we get,

$$\frac{1}{2} m \left(v_r^2 + v_\theta^2\right) - \frac{GmM}{R} = \frac{1}{2} m \left(\frac{R}{R+H}\right)^2 v_\theta^2 - \frac{GmM}{R+H} \quad \text{(iii)}$$

which corresponds to maximum height.

So, to the first order of H/R; the eqn (iii) becomes,

$$\frac{1}{2} m \left(v_r^2 + v_\theta^2\right) - \frac{GmM}{R} = \frac{1}{2} m \left(1+\frac{H}{R}\right)^{-2} v_\theta^2 - G\frac{mM}{R}\left(1+\frac{H}{R}\right)^{-1}.$$

$$\simeq \frac{1}{2} m \left(1 - 2\frac{H}{R}\right) v_\theta^2 - \frac{GmM}{R}\left(1-\frac{H}{R}\right).$$

i-e $$\frac{1}{2} m v_r^2 = -\frac{mH}{R} v_\theta^2 + \frac{GmMH}{R^2}$$

So, $$H = \frac{v_r^2 R}{2\left[\frac{GM}{R} - v_\theta^2\right]}$$

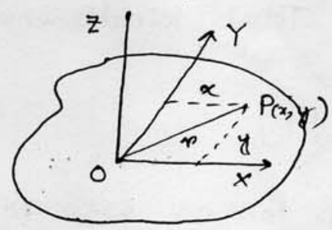
For vertical force $v_\theta = 0$; then

$$H = \frac{v_r^2}{2(GM/R^2)} = \frac{v_r^2}{2g}.$$

(6)

(a) Statement of perpendicular axes theorem:-

The moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it. [1]



Proof:- Let OZ be the axis that is perpendicular to the lamina passing through O and OX and OY are two perpendicular axes on the plane of the lamina.

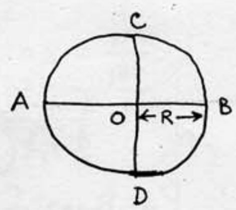
If we consider an elementary particle of mass m at P having coordinate x and y, then the distances of P from the axes OX and OY are y and x respectively. Hence,

$$I_x = \sum my^2 \quad \text{and} \quad I_y = \sum mx^2$$

$$\therefore I_x + I_y = \sum m(x^2+y^2) = \sum mr^2 = I_z \quad [\because x^2+y^2=r^2].$$

[1]

6 (b)



The moment of inertia of a circular disc about an axis perpendicular to its plane and passing through its centre is given by,

$$I = \frac{1}{2} MR^2$$

where M is the mass and R is the radius. [1]

Now let us consider two perpendicular diameters AB and CD of the circular disc. Since all the diameters are symmetrical, the moment of inertia of the disc about one diameter is the same as that about any other diameter.

If I_1 and I_2 are the moment of inertia of the disc about two axes perpendicular to each other, the applying the principle of perpendicular axes, the moment of inertia I of the disc about an axis perpendicular to the plane of the disc through O is given by,

$$I = I_1 + I_2 .$$ [2]

Since two diameters are symmetrical with respect to the disc, $I_1 = I_2$.

$$\therefore \; I = 2I_1 \quad \text{or} \quad I_1 = \frac{I}{2} = \frac{1}{2} \times \frac{1}{2} MR^2 = \frac{1}{4} MR^2 .$$ [1]

GROUP B

SECTION I

7 Answer any two questions :-

(a) The Young's modulus Y can be expressed as,

$$Y = \frac{T}{\pi r^2} \cdot \frac{1}{\text{Linear strain}}$$

where T = tension of the wire and r is the radius of the wire.

$$\therefore \text{ Linear strain} = \frac{T}{\pi r^2} \cdot \frac{1}{Y} = \frac{37.7}{3.14 \times (5 \times 10^{-4})^2} \cdot \frac{1}{2 \times 10^{11}} = 2.4 \times 10^{-4} .$$

Now the tension on the wire will just vanish, if the wire of length L be heated so that an elongation of length l occurs in its length L. If t be the required temperature in ^0C then,

$$\alpha = \frac{l}{L(t-20)} \quad \text{or} \quad t - 20 = \frac{2.4 \times 10^{-4}}{1.2 \times 10^{-5}} = 20 , \quad \text{So, } t = 40^\circ C .$$

⑦ (b)

Mass of gas before: $m_1 = \left(\frac{4}{3}\right)\pi r_1^3 \rho_1$

after: $m_2 = \left(\frac{4}{3}\right)\pi r_2^3 \rho_2$.

$$\therefore \frac{m_2}{m_1} = 2 = \frac{r_2^3 \rho_2}{r_1^3 \rho_1} = \frac{r_2^3 P_2}{r_1^3 P_1} = r_2^3\left(P + \frac{4T}{r_2}\right) \Big/ r_1^3\left(P + \frac{4T}{r_1}\right)$$

or $$2 r_1^3\left(P + \frac{4T}{r_1}\right) = r_2^3\left(P + \frac{4T}{r_2}\right)$$

P = Pressure and T = surface tension of soap bubble.

(c) We know deflection, $\delta = \dfrac{Wl^3}{(3YAK^2)}$

For circular cross section $A = \pi r^2$ and $K^2 = \dfrac{r^2}{4}$

For square cross section, $A = a^2$ and $K^2 = \dfrac{a^2}{12}$

Since A is same, so, $a^2 = \pi r^2$.

$$\therefore \frac{W_{circular}}{W_{square}} = \frac{(AK^2)_{circular}}{(AK^2)_{square}} = \frac{\pi r^2 \cdot (r^2/4)}{a^2 \cdot (a^2/12)} = \frac{3\pi r^4}{a^4}$$

$$= \frac{3\pi r^4}{\pi^2 r^4} = \frac{3}{\pi}.$$

(d)

Rate of decrease of volume $= -4\pi r^2 (dr/dt)$.

So,

$$-4\pi r^2 \frac{dr}{dt} = \frac{4T}{r} \frac{\pi a^4}{8\eta l}$$

or $$\int_{r_1}^{r_2} -r^3\, dr = \frac{Ta^4}{8\eta l}\int_0^t dt$$

or $$\frac{r_1^4 - r_2^4}{4} = \frac{Ta^4 t}{8\eta l} \quad \therefore t = \frac{2\eta l (r_1^4 - r_2^4)}{Ta^4}.$$

SECTION II

Answer any two questions. $6 \times 2 = 12$.

⑧ (a) Text book material.

(b) In this case the volume of the liquid flowing per second through the tube is given by,

$$V = \frac{\pi P a^4}{8\eta l}.$$

When the co-axial rod is inserted into the tube, the volume of the liquid flowing per second through the annular space is given by,

$$V' = \frac{P\pi}{8\eta l}(a^2 - b^2)\left[(a^2+b^2) - \frac{(a^2-b^2)}{\log\frac{a}{b}}\right]$$

Here $b = \frac{a}{\sqrt{2}}$.

$$\therefore\ V' = \frac{P\pi}{8\eta l}\left(a^2 - \frac{a^2}{2}\right)\left[\left(a^2+\frac{a^2}{2}\right) - \frac{\left(a^2-\frac{a^2}{2}\right)}{\log\sqrt{2}}\right]$$

$$= \frac{P\pi a^4}{8\eta l}\cdot\frac{1}{2}\left[\frac{3}{2} - \frac{1}{2\times 2\cdot 303\log\sqrt{2}}\right]$$

$$= \frac{P\pi a^4}{8\eta l}\cdot\frac{1}{2}\left[1\cdot 5 - 1\cdot 442\right] = 0\cdot 029\cdot\frac{P\pi a^4}{8\eta l}.$$

The rate of flow is reduced by,

$$V - V' = \frac{P\pi a^4}{8\eta l} - 0\cdot 029\,\frac{P\pi a^4}{8\eta l} = 0\cdot 971\,\frac{P\pi a^4}{8\eta l}$$

$\therefore$ The percentage of reduction of the rate of flow,

$$= \frac{V-V'}{V}\times 100 = 0\cdot 971\times 100 = 97\cdot 1\,\%.$$

(9) Text book material. Please consult any standard text book.

(10) Text book material. Please consult any standard text book

B.Sc (Hons) Examination – 2015 [P–1]

PHYSICS

PAPER – II

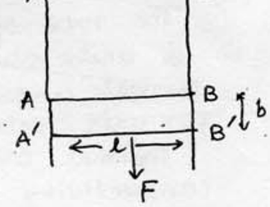
(Old Syllabus)

Model answers

GROUP–A

(1) Answer any four questions: $2\times4=08$

(a) When the area of liquid surface is increased, work is done against surface tension. This work is stored in the surface as surface energy.



Let a wire frame ABCD where AB is movable, is having a soap film over it. The wire AB is pulled inwards due to surface tension by a force $2T\times l$, where T is the surface tension and l is the length AB. The factor 2 appears because there are two surfaces. If the film is pulled by a short distance b to the position A′B′, keeping the temperature constant, then the work done $=2T\times l\times b$

Increase in area $=2\times l\times b$.

$\therefore$ Energy spent per unit area $=\dfrac{2Tlb}{2lb}=T$.

This energy is stored in the surface.

$\therefore$ The surface energy per unit area of a surface is numerically equal to the surface tension.

(b) The internal energy of an ideal gas is a function of temperature only. For free expansion of an ideal gas, we have

$$T\Delta S=\Delta Q=\Delta U+\Delta W=0+0=0.$$

Hence change in entropy should be zero.

(c) Here the weight of the air-bubble is neglected. Under this condition, Stoke's law takes the form,

$$6\pi\eta a v=\frac{4}{3}\pi a^3\rho g.$$

or $$v=\frac{2}{9}\frac{a^2\rho g}{\eta}.$$

Here $a=0.1$ cm, $\rho=1$ gm/c.c., $g=980$ cm/s^2 and $\eta=0.01$ Poise

Hence v = velocity of air bubble $=\dfrac{2}{9}\dfrac{(0.1)^2\times1\times980}{0.01}=217$ cm/s $=2.17$ m/S

(d)

For a twist θ, the couple required is given by $c = (n\pi r^4/2l).\theta$ where the symbols have their usual significance.

Work done for a further twist $d\theta = c\, d\theta = (n\pi r^4/2l)\theta \cdot d\theta$

$\therefore$ Total work done for the maximum twist θ_m from untwisted condition is,

$$\int_0^{\theta_m} c\, d\theta = \int_0^{\theta_m} \frac{n\pi r^4}{2l}\,\theta\, d\theta = \frac{n\pi r^4}{2l}\,\frac{\theta_m^2}{2}.$$

Now $C_m = (n\pi r^4/2l)\,\theta_m$.

$\therefore$ Total work done for twist θ_m = strain energy

$$= (C_m/\theta_m).(\theta_m^2/2) = \frac{1}{2} C_m . \theta_m .$$

(e)

The rate of increase of temperature in the variable state in a unit volume of the material is directly proportional to thermal conductivity and inversely proportional to thermal capacity per unit volume. The ratio of thermal conductivity to thermal capacity per unit volume is called thermometric conductivity.

Thus thermometric conductivity $= \frac{K}{\rho c}$.

Where K = Thermal conductivity & ρ and c are density and specific heat of the material respectively. [1]

Thermal conductivity is defined as the amount of heat transferred in unit time by conduction through a plane of unit area under a temperature gradient of unity, heat flowing perpendicular to the plane. [1]

(f)

We know for a Carnot engine, the efficiency is given by,

$$\eta = 1 - \frac{T_2}{T_1} \quad (T_2, T_1 \text{ temperatures of sink \& source}),$$

$$= (T_1 - T_2)/T_1 .$$

Therefore, for the same $T_1 - T_2$, η is less if T_1 is greater. Thus, increasing the temperature of the source would reduce η. It is therefore, more advantageous to reduce the temperature of the sink. In the limit when $T_2 \rightarrow 0$, $\eta \longrightarrow 1$.

(g)

We know, mean free path is given by,

$$\lambda = \frac{1}{(\sqrt{2}\, n \pi \sigma^2)}$$ where symbols have usual meanings.

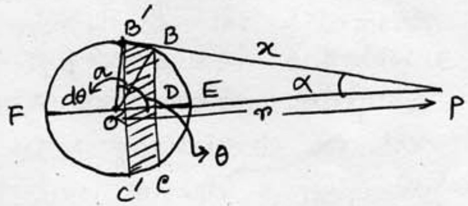
Now,
$$p = nKT \quad \text{or} \quad n = (p/KT)$$
$$= 25\times10^{24}\ m^{-3} \qquad [\text{Take } T = 293^\circ K = 273+20].$$
$$\therefore \lambda = \frac{1}{1.414 \times 25\times10^{24} \times 3.14 \times (2\times10^{-10})^2}$$
$$\simeq 2.25\times10^{-5}\ m.$$

GROUP—B

Answer any two: $12\times2 = 24$

(2)

(a)



We consider a thin spherical shell of radius a and surface density ρ. Let P be a point at a distance r from the centre O of the shell. We consider a slice $BB'CC'$ in the form of a ring with two planes close to each other and perpendicular to OP. So, the radius and the width of the rings are BD and BB' respectively. The area of such a slice is $(2\pi BD \times BB')$.

Now, $BD = a\sin\theta$ and $BB' = a\,d\theta$.

Therefore the mass of the slice $= 2\pi a \sin\theta\, d\theta . a . \rho$
$$= 2\pi a^2 \rho \sin\theta\, d\theta.$$

Hence the potential at P due to this slice
$$dV = -G\frac{2\pi a^2 \rho \sin\theta\, d\theta}{x}$$

Now from the triangle BOP, we have
$$BP^2 = OP^2 + OB^2 - 2\,OP.\,OB\cos\theta$$
$$\text{or} \quad x^2 = a^2 + r^2 - 2ar\cos\theta.$$

Differentiating we have,
$$2x\,dx = 2ar\sin\theta\, d\theta$$
$$\therefore \sin\theta\, d\theta = \frac{x\,dx}{ar}.$$

Therefore, substituting the value of $\sin\theta\, d\theta$, we have
$$dV = -G\frac{2\pi a \rho\, dx}{r}$$

Hence the potential for the whole shell can be obtained by integrating the above expression and is given by,
$$V = -\frac{2\pi a \rho G}{r}\int dx \qquad \text{——— (1)}$$

Now if the point P is on the surface of the shell, then, the limits of integration extend from $x = 0$ to $x = 2a$

$$\therefore \; V = -\frac{2\pi a \rho G}{r}\int_0^{2a} dx = -\frac{2\pi a \rho G}{r}\cdot 2a = -\frac{4\pi a^2 \rho G}{r} = -\frac{GM}{a}$$

Where we have taken, $M = 4\pi a^2 \rho$, the mass of the shell and $r = a$, when P is on the surface. [4]

For case (i), when the point is outside the shell. Here the limits of integration for eqn (1) extend from $x = (r-a)$ to $x = (r+a)$. Hence,

$$V = -\frac{2\pi a \rho G}{r}\int_{r-a}^{r+a} dx = -\frac{2\pi a \rho G}{r}\cdot 2a$$

$$= -\frac{4\pi a^2 \rho G}{r} = -\frac{GM}{r}.$$

Where, $M = 4\pi a^2 \rho$. This is the value of the potential that a point mass M situated at O would have produced at P. [1]

For case (ii), when the point is inside the shell, we shall take the limits of integration for eqn 1 as, $x = (a-r)$ to $x = (a+r)$.

$$\therefore \; V = -\frac{2\pi a \rho G}{r}\int_{a-r}^{a+r} dx = -\frac{2\pi a \rho G}{r}\cdot 2r = -\frac{4\pi a^2 \rho G}{a}$$

$$= -\frac{GM}{a}.$$

$\therefore$ The potential at any point inside the shell is constant and is equal to that on the surface. [1]

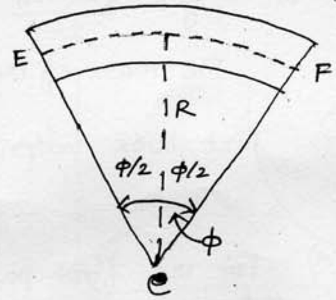
(b) The depression h of the mercury column as a result of surface tension will give rise to the error in the reading.

Now, $$h + \frac{r}{3} = \frac{2S\cos\theta}{r\rho g}.$$

$$\therefore \; h = \frac{2S\cos\theta}{r\rho g} - \frac{r}{3} = -0.3363 \text{ cm}.$$

Hence the error in reading will be 0.3363 cm.

(c)



Let us consider a portion of the bent beam and imagine the beam to be divided up into straight parallel longitudinal filaments. Before bending, all the filaments are of same length and the sections of this portion at the end are parallel.

But after bending, the two end faces are inclined to each other at an angle ϕ (say) as shown in the figure.

From the bending of beams it appears that, some of the filaments get shortened and some elongated and between them there is a surface, over which the filaments maintain their original length. This surface is called the <u>neutral surface</u> and the section of this surface in the plane of the figure is called <u>neutral axis</u> as shown by the dotted line. A straight line through C perpendicular to the plane of the figure is called <u>axis of bending</u>.

(3)

(a) We know, $\eta = \dfrac{\pi P a^4}{8 V l}$

where the symbols have usual meanings.

Putting the values of the quantities; we get, $\left[\text{Take } V = \dfrac{864}{12 \times 60}\ \text{c.c.}\right]$

$\eta = 0{\cdot}0114$ Poise.

$\therefore$ The maximum velocity of flow which is along the axis of the tube is,

$$v_{max} = \frac{Pa^2}{4\eta l} \quad (\because r = 0) \simeq 117\,\text{cm/s} \simeq 1{\cdot}17\,\text{m/s}.$$

Again critical velocity,

$$v_c = \frac{k\eta}{\rho a} \simeq 170\,\text{cm/s} = 1{\cdot}7\,\text{m/s}.$$

Thus $v_{max} < v_c$ and the condition for stream line motion exists.

(b)

When the pressure of CO_2 is 6 atmosphere and temperature $40^\circ C$, then we can write:

$$6V = \frac{30 \times 10^3}{44} \times \left(\frac{22{\cdot}4}{273}\right) \times 313 \quad \text{---} \quad (1)$$

and at the pressure of 4 atmosphere and $22^\circ C$,

$$4V = \frac{g}{44} \times \left(\frac{22{\cdot}4}{273}\right) \times 295 \quad \text{---} \quad (2)$$

$\therefore \; \frac{3}{2} = \frac{30\times 10^3}{g} \times \frac{313}{295}$

or $\quad g = \frac{626\times 10^4}{295}\, gm = \frac{6260}{295}\, kg \simeq 21.22\, kg.$

$\therefore$ The mass of the gas that leaked out $= 30 - 21.22 = 8.78\, kg$.

(c) Text book material. Please consult any standard text book.

(4)

(a) For the first part ~~plase~~ please consult any standard book for cylindrical heat flow.

The temperature distribution may have the form,

$$\theta = \frac{1}{\ln(r_2/r_1)}\left[(\theta_1 \ln r_2 - \theta_2 \ln r_1) - (\theta_1 - \theta_2)\ln r\right]$$

For the second part :—

The amount of heat flowing per second, Q, across (from inner to outer), the cylindrical isothermal surface of radius r and $r+dr$ is given by,

$$Q = -KA\frac{d\theta}{dr}$$

where A = area of conduction, K = thermal conductivity; θ = temperature at r and $\frac{d\theta}{dr}$ = temperature gradient which is negative.

Now we know, $\frac{d\theta}{dr} = \frac{(\theta_1 - \theta_2)}{\ln(r_1/r_2)}$ [Derived in the first part]

and $A = 2\pi r l$ $\quad l$ = length of the cylinder.

$$\therefore \; Q = -2\pi K l \frac{(\theta_1 - \theta_2)}{\ln(r_1/r_2)}$$

$$= 2\pi K l \frac{(\theta_1 - \theta_2)}{\ln(r_2/r_1)}$$

$$\therefore \; K = \frac{Q \ln(r_2/r_1)}{2\pi L(\theta_1 - \theta_2)}.$$

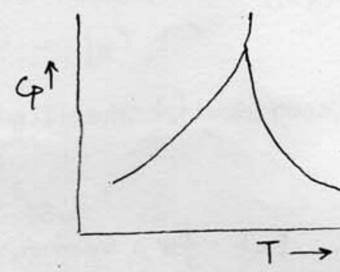
(b) (i) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ $\quad$ (ii) $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

(iii) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$ $\quad$ (iv) $\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$

[Please give $\frac{1}{2}$ marks for each relation]

5

(a) The transition between two liquid phases of ^4He, ordinary liquid He I and superfluid He II at 2.19K, is known as λ - transition.



A graph of C_p vs. T for the two phases of liquid He has the general shape a shown in the figure. This is similar or look alike to greek letter λ. Hence the name came λ - transition. [2]

(b) The distinguishing features between 1st order and λ - transition are;

(i) In 1st order phase transition, there is a change in entropy and volume but in λ - transition there is no change of entropy and volume. [1]

(ii) In 1st order phase transition, there is a transfer of latent heat but in λ - transition, no latent heat is absorbed. [1]

(b) We have from 1st Tds eqn,

$$Tds = C_v\, dT + T\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_{sat}$$

The suffix 'sat' indicates the condition of saturation is to be continued.

Now using Clasius - Clapeyron's eqn, we get

$$C_{sf} = C_v + \frac{L}{v_f - v_i}\left(\frac{\partial V}{\partial T}\right)_{sat}.$$

We have from 2nd Tds eqn,

$$Tds = dQ = C_p dT - T\left(\frac{\partial V}{\partial T}\right)_P dP.$$

Now specific heat of saturated vapour is,

$$C_{sf} = \left(\frac{dQ}{dT}\right)_{sat}$$

$$\therefore C_{sf} = C_{pf} - T\left(\frac{\partial V_f}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_{sat}$$

Again from Clausius - Clapeyron's eqn,

$$\left(\frac{\partial P}{\partial T}\right)_{sat} = \frac{L}{T(v_f - v_i)}$$

$$\therefore C_{sf} = C_{pf} - \frac{L}{T(v_f - v_i)}\left(\frac{\partial V_f}{\partial T}\right)_P \quad \text{---}(1)$$

Where m and M being the masses of rocket and earth.
So from (i) and (ii) we get,

$$\frac{1}{2} m (v_r^2 + v_\theta^2) - \frac{GmM}{R} = \frac{1}{2} m \left(\frac{R}{R+H}\right)^2 v_\theta^2 - \frac{GmM}{R+H} \quad \text{—(iii)}$$

which corresponds to maximum height.
So, to the first order of H/R; the eqn (iii) becomes,

$$\frac{1}{2} m (v_r^2 + v_\theta^2) - \frac{GmM}{R} = \frac{1}{2} m \left(1+\frac{H}{R}\right)^{-2} v_\theta^2 - \frac{GmM}{R}\left(1+\frac{H}{R}\right)^{-1}.$$

$$\simeq \frac{1}{2} m \left(1 - \frac{2H}{R}\right) v_\theta^2 - \frac{GmM}{R}\left(1-\frac{H}{R}\right).$$

i-e $$\frac{1}{2} m v_r^2 = -\frac{mH}{R} v_\theta^2 + \frac{GmMH}{R^2}$$

So, $$H = \frac{v_r^2 R}{2\left[\frac{GM}{R} - v_\theta^2\right]}$$

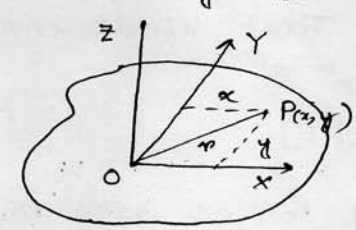
For vertical force $v_\theta = 0$; then

$$H = \frac{v_r^2}{2(GM/R^2)} = \frac{v_r^2}{2g}.$$

(6)

(a) Statement of perpendicular axes theorem:-
The moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it. [1]

Proof:- Let OZ be the axis that is perpendicular to the lamina passing through O and OX and OY are two perpendicular axes on the plane of the lamina.



If we consider an elementary particle of mass m at P having coordinate x and y, then the distances of P from the axes OX and OY are y and x respectively. Hence,

$$I_x = \sum my^2 \quad \text{and} \quad I_y = \sum mx^2$$

$$\therefore I_x + I_y = \sum m(x^2+y^2) = \sum mr^2 = I_z \quad [\because x^2+y^2 = r^2].$$

[1]

GROUP-C

Answer any three taking at least one from each section.

$6 \times 3 = 18$

SECTION I

(6) Text book material. Please consult any standard text book.

(7)

(a) $m_1 = \left(\frac{4}{3}\right)\pi r_1^3 \rho_1$ and $m_2 = (4/3)\pi r_2^3 \rho_2$.

$\therefore \frac{m_1}{m_2} = 2 = \frac{r_2^3 P_2}{r_1^3 P_1} = r_2^3\left(P + \frac{4T}{r_2}\right) \Big/ r_1^3\left(P + \frac{4T}{r_1}\right)$

$\therefore 2r_1^3\left(P + \frac{4T}{r_1}\right) = r_2^3\left(P + \frac{4T}{r_2}\right)$.

(b) At a distance r from the axis of the tube, the velocity v of the liquid is given by,

$$v = -\frac{Pr^2}{8\eta l} + C_1 \log r + C_2$$ [1]

Now, $v = 0$ at $r = b$ and $r = a$. Applying these conditions, we have,

$$C_1 = \frac{P(a^2 - b^2)}{4\eta l \log(a/b)} \text{ and } C_2 = \frac{Pa^2}{4\eta l} - \frac{P(a^2-b^2)\log a}{4\eta l \log(a/b)}$$

Substituting the values of C_1 and C_2 we have,

$$v = \frac{P}{4\eta l}\left[(a^2 - r^2) + \frac{(a^2-b^2)\log(r/a)}{\log(a/b)}\right]$$ [2]

$\therefore$ Volume of the liquid flowing per second through the annular space,

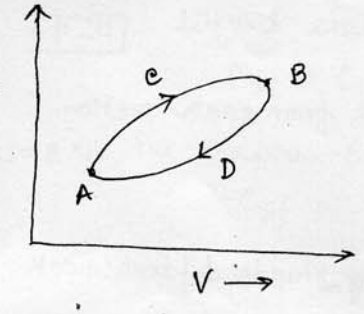
$$V = \int_b^a 2\pi v r \, dr.$$

Putting the value of v and integrating, we obtain,

$$V = \frac{P\pi}{8\eta l}(a^2 - b^2)\left[(a^2+b^2) - \frac{(a^2-b^2)}{\log(a/b)}\right].$$ [1].

SECTION-II

(8) Whenever a system passes from an initial state to a final state its entropy changes. The change in entropy solely depends on its value at the above two states but not on the ways through which the change takes place. From this point of view entropy is called a state function. This is explained by the following example.



The points A & B represents the two states of a system on the indicator diagram. The system passes from A to B through C and returns to A via D. This constitute a reversible cycle. The change in entropy may be interpreted as,

$$ds = {}_{C}\int_A^B \frac{dQ}{T} + {}_{D}\int_B^A \frac{dQ}{T}.$$

Since the physical change from B to A will be exactly equal and opposite corresponding to its reversed path A to B, therefore we may interprete,

$$-{}_{D}\int_A^B \frac{dQ}{T} = {}_{D}\int_B^A \frac{dQ}{T}. \qquad [2]$$

So,

$$ds = {}_{C}\int_A^B \frac{dQ}{T} - {}_{D}\int_A^B \frac{dQ}{T} = 0.$$

$$\therefore \; {}_{C}\int_A^B \frac{dQ}{T} = {}_{D}\int_A^B \frac{dQ}{T}.$$

∴ The change in entropy does not depend on the way through which change occurs i.e entropy is a state function.

[1]

(b)

Change in entropy when m gm of ice at T_1K changes to water at T_1 K is $\Delta S_1 = \frac{mL_1}{T_1}$ where L_1 = Latent heat of fusion of ice per gram.

Change in entropy, when m gm of water at T_1 K is raised to a temperature T_2K is,

$$\Delta S_2 = \int_{T_1}^{T_2} ds = \int_{T_1}^{T_2} \frac{mc\,dT}{T} = mc\ln\left(T_2/T_1\right). \qquad [2]$$

Where c is the specific heat of water.

Lastly, change in entropy when m gm of water at T_2K is converted into steam at T_2K is,

$\Delta S_3 = \frac{mL_2}{T_2}$ where L_2 is the latent heat of vapourization of water per gram.

∴ Total change in entropy

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3$$

$$= \frac{mL_1}{T_1} + mc\ln\left(T_2/T_1\right) + \frac{mL_2}{T_2}. \qquad [1]$$

(9)

(a) If we express the adiabatic and isothermal bulk modulus of elasticity as E_S and E_T, we get the ratio as,

$$\frac{E_S}{E_T} = \frac{-v\left(\frac{\partial P}{\partial V}\right)_S}{-v\left(\frac{\partial P}{\partial V}\right)_T} = \frac{\left(\frac{\partial P}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial V}\right)_T} = \frac{\left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial T}{\partial V}\right)_S}{\left(\frac{\partial P}{\partial S}\right)_T \left(\frac{\partial S}{\partial V}\right)_T} \quad [1]$$

Applying Maxwell's relation we get,

$$\left(\frac{\partial P}{\partial T}\right)_S = \left(\frac{\partial S}{\partial V}\right)_P \quad ; \quad \left(\frac{\partial P}{\partial S}\right)_T = -\left(\frac{\partial T}{\partial V}\right)_P .$$

$$\therefore \quad \left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V \quad \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V . \quad [1]$$

So putting these values we get,

$$\frac{E_S}{E_T} = \frac{-\left(\frac{\partial S}{\partial V}\right)_P \left(\frac{\partial P}{\partial S}\right)_V}{-\left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_V} = \frac{\left(\frac{\partial S}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial S}\right)_V}{\left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial P}{\partial T}\right)_V}$$

$$= \left(\frac{\partial S}{\partial T}\right)_P \left(\frac{\partial T}{\partial S}\right)_V$$

$$= \frac{T\left(\frac{\partial S}{\partial T}\right)_P}{T\left(\frac{\partial S}{\partial T}\right)_V} = \frac{C_P}{C_V} = \gamma . \quad [2]$$

(b)

We have from Clausius – Clapeyron equation,

$$\left(\frac{dP}{dT}\right)_{12} = \frac{L_{12}}{T(V_2 - V_1)} .$$

Where, V_2 = specific volume of water at $0^\circ C$ = 1 c.c.

V_1 = specific volume of ice at $0^\circ C = \left(1 + \frac{9.1}{100}\right)$ c.c.

$= 1.091$ c.c.

$L_{12} = 80\ \text{cal/gm} = 80 \times 4.2 \times 10^7\ \text{ergs/gm}$.

$T = 0^\circ C = 273\ K$.

$$\therefore \quad \left(\frac{dP}{dT}\right)_{12} = \frac{80 \times 4.2 \times 10^7}{273 \times (-0.091)} = -13.52 \times 10^7 \text{ dynes cm}^{-2} K^{-1}.$$

The –ve sign indicates that melting point of ice is lowered by the increase of pressure.

(10)

(a) We have for the coefficient of viscosity of a gas;

$$\eta = \frac{1}{3} m N \langle c \rangle L = \frac{1}{3} \rho \langle c \rangle L \quad \text{---} (1)$$

where $mN = \rho$ = density of the gas; L = mean free path and $\langle c \rangle$ is the mean speed of the gaseous molecules.

Now, $L = \dfrac{1}{\sqrt{2}\pi\sigma^2 N} = \dfrac{m}{\sqrt{2}\pi\sigma^2\rho}$ $\quad [\because mN = \rho]$

For a given gas; m and σ are constants.

$\therefore \rho L =$ constant ———— (2)
at a particular temperature.

From (1) and (2); we see that the co-efficient of viscosity must be independent of ~~pressure~~. density at a particular temperature. [It is also known as Maxwell's Law].

(b)

For thickness dx to grow, when the thickness is already x, in time dt, $L\rho\, dx$ calorie must be transmitted across the area of layer x.

$\therefore L\rho\, dx = \dfrac{K\theta}{x} dt$; where $\dfrac{\theta}{x}$ = temperature gradient

$\therefore$ Rate of growth of thickness;

$$\frac{dx}{dt} = \frac{K\theta}{L\rho x} = \frac{2.184 \times (273-261)}{333 \times 10^3 \times 920 \times 0.04} \text{ m/s}$$

$$= 7.69 \times 10^{-3} \text{ m/h}.$$