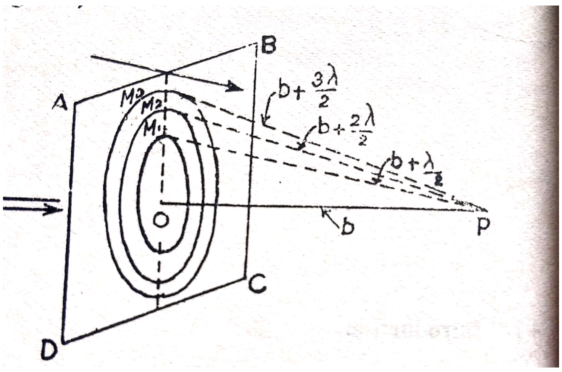
SEM-II
Hons (C IV: WAVES AND OPTICS)
L-7
Manoj Kumar Saha,
Assistant Professor, Department of Physics K C College Hetampur

Explanation of Rectilinear Propagation of Light, Theory of a Zone Plate:

Explanation of Rectilinear Propagation of Light:



We have just seen that light reaching p from the wave front $ABCD$ is equivalent to the contribution from half the first period zone. i.e. $\frac{\pi b\lambda}{2}$. Since λ is very small $\pi b\lambda$ is very small. If a circular disc of area $\pi b\lambda$ obstruct light to reach from the first half period zone the amplitude at p from the remaining wave front is $\frac{d_2}{2}$ and the intensity is proportional to $(\frac{d_2}{2})^2$. If now the size of the obstructing disc be increased gradually so as to cover the first two, three, four half period zones the resultant intensity p will be proportional to

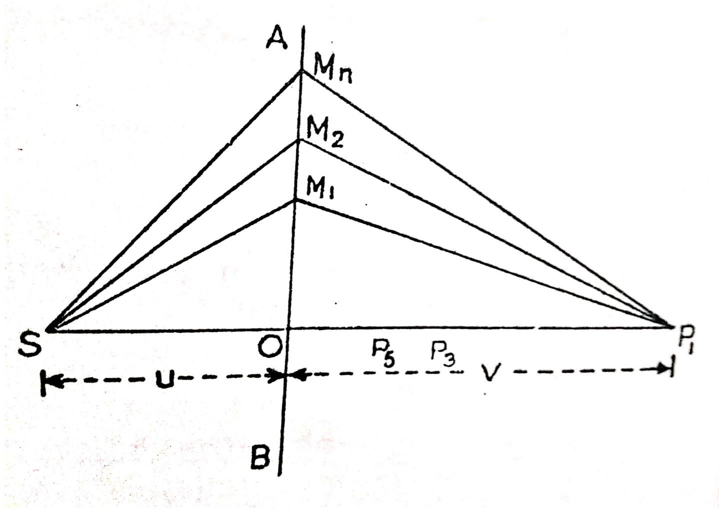
$$(\frac{d_3}{2})^2, \ (\frac{d_4}{2})^2, \ (\frac{d_5}{2})^2$$

respectively. As $d_1, \ \ d_2, \ d_3$ gradually decrease the resultant intensity at p will gradually as the area of the disc obstructing light is gradually increase. If the size of the obstacle is sufficiently large to cover a number of half period zones, the point P will be almost dark. But the wavelength of light being small, a small obstacle is sufficient to cover a large number of half period zones. So any point beyond that obstacle remains dark. This explains for the approximate rectilinear propagation of light.

Theory of a Zone Plate:

We have just considered that according to the Fresnel's theory of half period zones, light from any two consecutive zones reaches at the point under consideration in opposite phase. As a result the intensity at the point is greatly reduced. If we make the alternate zones opaque, the waves reaching the point will reinforce each other producing intense illumination. This principle is applied in the construction of a zone plate. Zone plate is simply a plane parallel glass plate having concentric circles of radii proportional to the square root of natural numbers $1, \ 2, \ 3, \ 4$etc. Then even or odd order annular space between the circle are made opaque.

Let us suppose that a plane monochromatic light wave for which the plate is constructed is incident normally on it.on the other side on its axis we shall come across a series of maxima points having intensities in the increasing order as the distance of the points from the plate is increases. The zone plate acts therefore like a convergent lens but unlike a lens it has a series of foci and associated focal lengths.



Theory

Let AB be the zone plate normal to the plane of the paper, S is a source of monochromatic light of wavelength λ. Let us find the amplitude of the light wave at a point p_4 on the other side of the plate and on its axis. Let $os = U$ and $op_1 = V$ and mark off points M_1 , M_2 , $M_3, ., ., ., M_n$ on the plate so that

$$\begin{aligned} SO + OP_1 + \tfrac{\lambda}{2} &= SM_1 + M_1P_1 \\ SO + OP_1 + \tfrac{2\lambda}{2} &= SM_2 + M_2P_1 \\ SO + OP_1 + \tfrac{n\lambda}{2} &= SM_n + M_nP_1 \end{aligned} \tag{1}$$

putting the value of U, V we get

$$SM_n + M_nP_1 = U + V + \tfrac{n\lambda}{2}$$

Let the radii of the half period zones be

$$OM_1 = r_1, OM_2 = r_2, ..., OM_n = r_n$$

Therefore

$$SM_n^2 = (U^2 + r_n^2)^{\frac{1}{2}} = U[1 + (\frac{r_n}{U})^2]^{\frac{1}{2}} \simeq U[1 + \frac{1}{2}(\frac{r_n}{U})^2] \tag{2}$$

Similarly

$$P_1M_n \simeq V[1 + \frac{1}{2}(\frac{r_n}{V})^2] \tag{3}$$

Therefore from(1), (2) and (3) we have

$$\frac{r_n^2}{2U} + \frac{r_n^2}{2V} = \frac{n\lambda}{2} \tag{4}$$

or

$$r_n^2 = \frac{UV}{U+V} n\lambda \tag{5}$$

Putting $n = 1, 2, 3$etc we get the value of r_1, r_2, r_3. These radii of the half period zones are proportional to the square root of the natural numbers. we have mentioned that zone plate is constructed by blacking the alternate zones. If the 2nd 4th 6th .. etc zones are intercepted the resultant amplitude at p_1 will be $D = d_1 + d_3 + d_5 + ...$ this amplitude is grater than $\frac{d_1}{2}$.

The behavior of zone plate is then similar to the convex lens. From (2) the relation between $U$and V is given by

$$\frac{1}{U} + \frac{1}{V} = \frac{n\lambda}{r_n^2} \tag{6}$$

The equivalent focal length $f = \frac{r_n^2}{n\lambda}$