SEM-IV
PHYH-C IX: ELEMENTS OF MODERN PHYSICS

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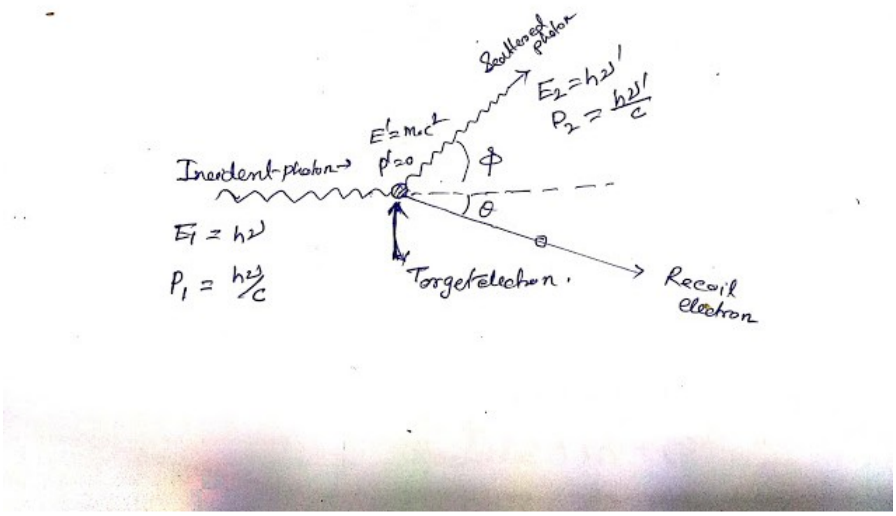
COMPTON EFFECT

When a photon of high energy $h\nu$ collide with a free electron of the scatter (target electron) at rest, it transfer some energy to the electron .The scattered photon will therefore have a smaller energy $h\nu'$ and consequently a grater wavelength than that of the incident photon. The observed change in wavelength of the scattered photon by considering the elastic collision between the incident photon and the free electron of the scattered material is known as$\underline{Compton effect}$.



Let us now consider a photon of energy $h\nu$ collide with an free electron of the scatter. The photon is scattered at an angle ϕ, while the electron recoils at an angle θ, which shown in the figure.
Now applying the law of momentum conservation along and perpendicular to the direction of incidence, we get

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c} Cos\phi + pCos\theta \tag{0.1}$$

$$0 = \frac{h\nu'}{c} Sin\phi + pSin\theta \tag{0.2}$$

Where p is the momentum of the recoil electron.

$$pcCos\theta = h\nu - h\nu' Cos\phi \tag{0.3}$$

$$pcSin\theta = h\nu - h\nu' Sin\phi \tag{0.4}$$

Squaring and adding we get,

$$p^2c^2 = h\nu^2 - 2h\nu h\nu' Cos\phi + (h\nu')^2 \tag{0.5}$$

Now, applying the law of energy conservation
E"=K.E. of recoil electron+its rest mass energy

$$= h\nu - h\nu' + m_0c^2 \tag{0.6}$$

Again for a Relativistic recoil electron

$$E'' = \sqrt{p^2c^2 + m_0^2c^4} \tag{0.7}$$

So we can write

$$p^2c^2 + m_0^2c^4 = [h\nu - h\nu' + m_0c^2]^2 \tag{0.8}$$

After some simple calculation we can get

$$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - Cos\phi) \tag{0.9}$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - Cos\phi) \tag{0.10}$$

$$\underline{\lambda' - \lambda = \frac{h}{m_0c^2}(1 - Cos\phi)} \tag{0.11}$$

This equation(0.13) gives the change (increase) in wavelength or Compoton shift of scattered photon by a particle of rest mass m_0.

Compton wavelength:
Since the maximum value of $Cos\phi = 1$, wavelength of the scattered photon is always grater than the incident photon. This change is independent of the wavelengthλ of the incident photon and the quantity $\frac{h}{m_0c} = \lambda_c$ is known as $\underline{Comptonwavelength}$ of the scattering particle.
From equation(0.13) we can conclude
1.When$\phi = 0$, then $\lambda' - \lambda = 0$ i.e there is no scattering along the direction of incident photon.

2.When$\phi = \frac{\pi}{2}$, then $\lambda' - \lambda = \frac{h}{m_0c} = \lambda_c =$ Compton wavelength.

$\lambda_c = \frac{h}{m_0c} = 0.2427A^0$
Compton shift $\delta\lambda = 0.2427(1 - Cos\phi)$

3..When$\phi = 180^0$, then $\lambda' - \lambda = 2\frac{h}{m_0c}$i. e. wavelength change will be twice of the Compton wavelength λ_c and this is the maximum possible shift.

Find the Direction of Recoil Electron:

electron

Now dividing equation 6 by 5

$$\tan\theta = \frac{h\nu'\sin\phi}{h\nu - h\nu'\cos\phi}$$

$$= \frac{\nu'\sin\phi}{\nu - \nu'\cos\phi} \quad \text{——— (14)}$$

Then from equation (12) we can write.

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0c^2}(1-\cos\phi)$$

or $$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0c^2}(1-\cos\phi)$$

or $$\frac{\nu}{\nu'} = 1 + \frac{h\nu}{m_0c^2}\cdot 2\sin^2\phi/2$$

So, the frequency of scattered photon

$$\nu' = \frac{\nu}{1+\frac{h\nu}{m_0c^2}\cdot 2\sin^2\phi/2}$$

Then from equation (13)

$$\tan\theta = \frac{\frac{\nu\sin\phi}{1+2\alpha\sin^2\phi/2}}{\left[\nu - \left(\frac{\nu}{1+2\alpha\sin^2\phi/2}\right)\cos\phi\right]}, \quad \text{where } \alpha = \frac{h\nu}{m_0c^2}$$

$$\therefore \tan\theta = \frac{\sin\phi}{1+2\alpha\sin^2\phi/2 - \cos\phi} = \frac{2\sin\phi/2\cdot\cos\phi/2}{2\alpha\sin^2\phi/2 + (1-\cos\phi)}$$

$$= \frac{2\sin\phi/2\cdot\cos\phi/2}{2\sin^2\phi/2 + 2\alpha\sin^2\phi/2} = \frac{\sin\phi/2\cdot\cos\phi/2}{\sin^2\phi/2(1+\alpha)} = \frac{\cot\phi/2}{1+\alpha}$$

$$= \frac{\cot\phi/2}{1+\frac{h\nu}{m_0c^2}}$$

This gives the direction of the recoil electron in terms of direction and frequency of the scattered photon.

Problem-1 In Compton scattering experiment a beam of γ radiation having wavelength of photon $2.426 \times 10^{-12}m$ is incident on a foil of aluminum.

a. Find the Compton shift , for the scattering angle 45^0.

b. Find the wavelength of scattered radiation if the scattered radiation are view at an angle 45^0 to the direction of the incident beam.
c. Find the Compton wavelength.

d. What is the energy of the incident photon?

e. What is the energy of the scattered photon?

f. Find the energy lost by the photon?

g. Find how much K.E. is imparted to the recoil electron?

h. Find the direction of emission of the corresponding recoil electron(assuming scattered angle $\phi = 90^0$) .

i. Find the total energy of the recoil electron .

j. Find the fraction of energy loss.

Problem-2. Compton effect can not be observed with visible light-Why?

The wavelength of visible light is $\lambda = 6000 \times 10^{-10} m$

Energy of the visible light

$E = \frac{hc}{\lambda} = \frac{6.627 \times 10^{-34} \times 3 \times 10^8}{6000 \times 10^{-10} \times 1.6 \times 10^{-19}}$

$= 2eV$

The binding energy of an electron in the atoms near $10eV$. For example the binding energy of an electron in hydrogen atom $= 13.6eV$.
So when visible light fall on a target it can not liberate electrons of the scatterer. So we can not see Compton effect with visible light.