Solve

| | A | B | C | D | E | $a_i$ |
|---|---|---|---|---|---|---|
| X | 55 | 30 | 40 | 50 | 40 | 40 |
| Y | 35 | 30 | 100 | 45 | 60 | 20 |
| Z | 40 | 60 | 95 | 35 | 30 | 40 |
| $b_j$ | 25 | 10 | 20 | 30 | 15 | |

Determine the optimal distribution to minimize the cost of transportation.

Solⁿ: Obviously $\sum a_i = \sum b_j = 100$, so it is a balanced T.P. Now we solve it by VAM —

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\boxed{\phantom{0}}$ 55 | $\boxed{10}$ 30 | $\boxed{20}$ 40 | 50 | $\boxed{10}$ 40 | 40 (10) | 20(10) | 10(10) | 10(10) | 10(10) |
| $\boxed{20}$ 35 | 30 | 100 | 45 | 60 | 20(5) | 20(5) | 20(10) | | |
| $\boxed{5}$ 40 | 60 | 95 | $\boxed{30}$ 35 | $\boxed{5}$ 30 | 40(5) | 40(5) | 40(5) | 40(5) | 35(5) |
| 25 (5) | 10 (0) | 20 (55) | 30 (10) | 15 (10) | | | | | |
| 25 (5) | 10 (0) | | 30 (10) | 15 (10) | | | | | |
| 25 (5) | | | 30 (10) | 15 (10) | | | | | |
| 5 (15) | | | 30 (15) | 15 (10) | | | | | |
| | | | 30 (15) | 15 (10) | | | | | |

No. of initial basic cells = 7 $(m+n-1)$; so the problem is a non-degenerate one.

Now we use UV-method to find optimal solⁿ.

| | A | B | C | D | E | |
|---|---|---|---|---|---|---|
| X | $\boxed{-5}$ 55 | $\boxed{10}$ 30 | $\boxed{20}$ 40 | $\boxed{-5}$ 50 | $\boxed{10}$ 40 | $u_1=0$ |
| Y | $\boxed{20}$ 35 | $\boxed{-15}$ 30 | $\boxed{-75}$ 100 | $\boxed{-15}$ 45 | $\boxed{-35}$ 60 | $u_2=-15$ |
| Z | $\boxed{5}$ 40 | $\boxed{-40}$ 60 | $\boxed{-65}$ 95 | $\boxed{30}$ 35 | $\boxed{5}$ 30 | $u_3=-10$ |
| | $v_1=50$ | $v_2=30$ | $v_3=40$ | $v_4=45$ | $v_5=40$ | |

since all non basic cell evaluations are negetive so optimal solution has been reached. The solution will be $x_{12}=10$; $x_{13}=20$; $x_{15}=10$

$x_{21}=20$; $x_{31}=5$; $x_{34}=30$; $x_{35}=5$.

and min cost $= (10\times30)+(20\times40)+(10\times40)+(20\times35)+(5\times40)+(30\times35)+(5\times30)$ units.

$= 300+800+400+700+200+1050+150$ units

$= 3600$ units.

Ans

Solve

UNBALANCED T.P

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | capacity |
|---|---|---|---|---|---|
| $F_1$ | 15 | 24 | 11 | 12 | 5000 |
| $F_2$ | 25 | 20 | 14 | 16 | 4000 |
| $F_3$ | 12 | 16 | 22 | 13 | 7000 |
| Req. | 3000 | 2500 | 3800 | 4000 | |

Find the least cost transportation schedule.

Soln:- In this problem $\sum a_i = 5000 + 4000 + 7000$
$= 16000$.

and $\sum b_j = 3000 + 2500 + 3500 + 4000 = 13000$.

Since $\sum a_i \neq \sum b_j$ so it is an unbalanced T.P again $\sum a_i > \sum b_j$ and so a dummy column $W_5$ should be introduced with requirement 3000 to make this problem a balanced T.P. The corresponding balanced T.P is shown below.

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | $a_i$ |
|---|---|---|---|---|---|---|
| $F_1$ | 15 | 24 | 11 | 12 | 0 | 5000 |
| $F_2$ | 25 | 20 | 14 | 16 | 0 | 4000 |
| $F_3$ | 12 | 16 | 22 | 13 | 0 | 7000 |
| $b_j$ | 3000 | 2500 | 3500 | 4000 | 3000 | |

Now we solve the above by VAM for an initial basic feasible solution...

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $F_1$ | 15 | 24 | $\boxed{2500}$ 11 | $\boxed{2500}$ 12 | 0 | 5000 (11) | 5000 (1) | 5000 (1) | 5000 (1) | 5000 (1) | 2500 |
| $F_2$ | 25 | 20 | $\boxed{1000}$ 14 | 16 | $\boxed{3000}$ 0 | 4000 (14) | 1000 (2) | 1000 (2) | 1000 (2) | 1000 (2) | 1000 |
| $F_3$ | $\boxed{3000}$ 12 | $\boxed{2500}$ 16 | 22 | $\boxed{1500}$ 13 | 0 | 7000 (12) | 7000 (1) | 4500 (1) | 1500 (0) | | |
| | 3000 (3) | 2500 (4) | 3500 (3) | 4000 (1) | 3000 (0) | | | | | | |
| | 3000 (3) | 2500 (4) | 3500 (3) | 4000 (1) | | | | | | | |
| | 3000 (3) | | 3500 (3) | 4000 (1) | | | | | | | |
| | | | 3500 (3) | 4000 (1) | | | | | | | |
| | | | 3500 (3) | 2500 (4) | | | | | | | |
| | | | 3500 | | | | | | | | |

Number of basic cells = 7 = (5+3−1) so, the problem is a non-degenerate one.

Now we use UV method to get an optimal soln.

(Check it)

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | |
|---|---|---|---|---|---|---|
| $F_1$ | [-4] 15 | [-9] 24 | 2500 11 | 2500 12 | [-3] 0 | $u_1 = -1$ |
| $F_2$ | [-11] 25 | [-2] 20 | 1000 14 | [-1] 16 | 3000 0 | $u_2 = +2$ |
| $F_3$ | 3000 12 | 2500 16 | [-10] 22 | 1500 13 | [-2] 0 | $u_3 = 0$ |
| | $V_1 = 12$ | $V_2 = 16$ | $V_3 = 12$ | $V_4 = 13$ | $V_5 = -2$ | |

Since all non basic cell evaluations are negetive, so the optimal solution has been reached and the solution will be —

$x_{13} = 2500$; $x_{14} = 2500$; $x_{23} = 1000$; $x_{31} = 3000$

$x_{32} = 2500$ $x_{34} = 1500$

minimum cost $= (2500 \times 11) + (2500 \times 12) + (1000 \times 14) + (3000 \times 12) + (2500 \times 16) + (1500 \times 13)$ unit

$= 167000$ unit.

## B) Loop in a Transportation Table

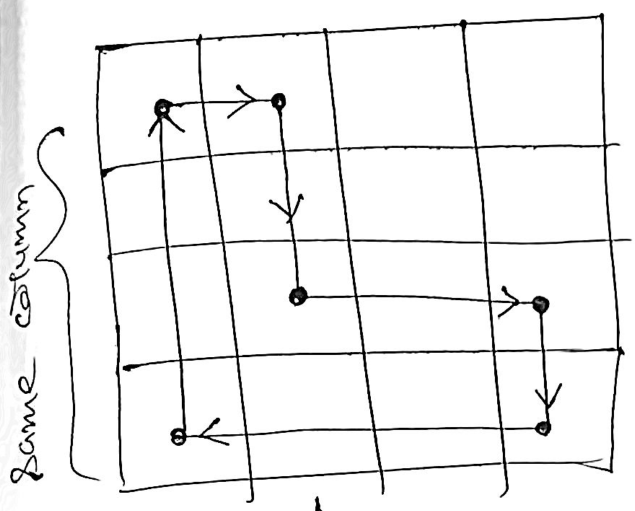
In a transportation table, an ordered set of four or more cells are said to be form a loop :

(i) Iff two consecutive cells in the ordered set lie either in the same row or in the same column.

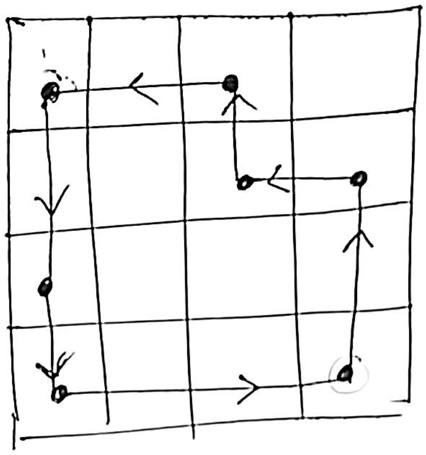
(ii) The 1st and last cell also lie either in the same row or in the same column.

lie 1st and last cell same row

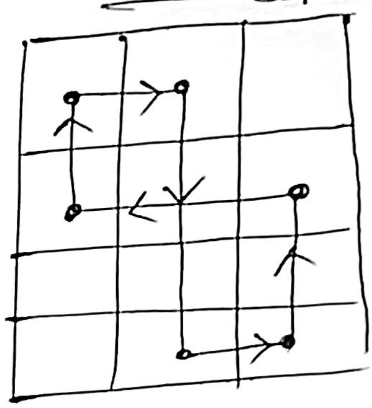
lie 1st and last cell same column




$L_1$



$L_2$



$L_3$

the ordered set of cells in the circuit are

$L_1 = \{(1,1), (1,2), (3,2), (3,4), (4,4), (4,1)\}$

$L_2 = \{(1,1), (3,1), (4,1), (4,4), (2,4), (2,3), (1,3)$

$L_3 = \{(1,1), (1,2), (4,2), (4,3), (2,3), (2,1)\}$