B.Sc. Part-III (Honours) Examination, 2016

Subject : Physics

Paper-IX

Time: 4 Hours **Full Marks: 100**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group A

1. Answer *any eight* questions from the following: $2\times8=16$

(a) Illuminating the surface of a certain metal alternately with light of wavelengths λ_1 and λ_2 it was found that the maximum velocity of the corresponding photoelectrons decreases by a factor of 2. Find an expression of the work function of the metal in terms of λ_1, λ_2, Planck's constant and the velocity of light.

(b) For the case of hydrogen atom, find the wavelength of the first line of the Balmer series in nano-metre.

(c) Calculate the difference between ionisation potentials of atomic hydrogen and deuterium. (Assume $m_D = 2\ m_H$)

(d) What are the characteristics of the two events if their interval is (i) time-like, (ii) space-like?

(e) Show that proper time is Lorentz invariant.

(f) The energy of a particle confined in a one-dimensional box of length L cannot be zero. Justify this statement using the uncertainty principle.

(g) Show that $[L^2, L_z] = 0$, where L and L_z are the angular momentum and its z component of the electron in hydrogen atom.

(h) Show that the probability of finding the electron in a hydrogen atom is maximum at Bohr radius (a). [The ground state wave function of hydrogen atom is given as $\psi_0 = Ae^{-r/a}$. Here A is normalization constant.]

(i) Explain, in terms of the centrifugal potential of the hydrogen atom, why the radial wave functions of the 's' states are maximum at r = 0.

(j) All nuclei with Z > 83 and A > 209 spontaneously decay into lighter nuclei with the emission of alpha particles. Explain this phenomenon with the help of 'saturation' of nuclear forces.

(k) How does the shell model account for the stability of the ^{16}O nucleus?

(l) How can you explain the continuous energy spectrum of electrons in the case of β-decay of a nucleus?

(m) Write two properties of a nucleus which can be compared to that of a liquid drop.

(n) Write the spin and parity quantum numbers of the 'p_i' and 'rho' mesons.

Group B

Answer *any four* questions, taking at least *one* from each section. $12\times4=48$

Section - I

2. (a) Compton scattering takes place between a photon of wavelength λ and an electron of mass m, which is at rest in the laboratory frame. If λ' is the wavelength of the photon after the scattering, find an expression of $(\lambda' - \lambda)$ in terms of the scattering angle ϕ.

(b) If the value of λ' with $\phi = 120°$ is twice of that with $\phi = 60°$, what is value of λ in nano-metre?

(c) Find the value of Compton wavelength. 8+3+1=12

3. (a) What was the purpose of the Stern-Gerlach experiment? Why is a non-uniform magnetic field used in this experiment?

(b) Draw a simple diagram of the arrangement of the Stern-Gerlach experiment, including the observation on the photographic plate.

(c) Elucidate the significance of the experimental observation. What should the observation be as per the expectation in classical physics. Give reason for your answer. (1+1)+5+(3+1+1)=12

4. (a) Write briefly the motivation behind the Michelson-Morley experiment.

(b) Assuming the existence of ether and with the help of necessary diagrams, obtain an expression of the fringe-shift ΔN as a result of the rotation of Michelson's interferometer by 90°.

(c) What value of ΔN was observed in the experiment? What is the implication of the observation? 2+8+(1+1)=12

Section - II

5. A one-dimensional square potential barrier of height V_0 and length a is defined by,

$$
\begin{aligned}
V_0(x) &= 0 \text{ for } x < 0 \\
&= 0 \text{ for } x > a \\
&= V_0 \text{ for } 0 \leq x \leq a
\end{aligned}
$$

Find expressions for reflection and transmission coefficients for $E < V_0$. Hence plot transmission coefficient with energy of the particle. 5+5+2=12

6. (a) Write the Schrödinger equation for the hydrogen atom in spherical polar coordinates, with the coulomb potential.

(b) Separate the equation in (a) by standard procedure into three equations corresponding to the coordinates r, θ, φ.

(c) What are the possible values of the l and m_l quantum numbers when the principal quantum number is n = 3?

$$\left[\text{Given, in spherical polar coordinates,} \quad \nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \phi^2} \right].$$

2+(3+3+2)+2=12

Section - III

7 Construct the Bethe-Weizsacker mass formula of a nucleus in the framework of the liquid drop model taking into account,

(i) volume energy,

(ii) surface energy,

(iii) coulomb energy,

(iv) asymmetry energy and

(v) pairing energy.

Show the mass parabola in a diagram for even-A and odd-A nuclei and justify. 9+3=12

8. (a) Draw a simple diagram of the Geiger-Muller counter and briefly describe its various parts.

(b) Draw the characteristic curve for the Geiger-Muller counter showing the plateau region. Briefly explain how the shape of the curve arises.

(c) What is the necessity of quenching? Describe the quenching process in a self-quenching counter. (2+2)+(1+3)+(1+3)=12

9. (a) Regarding nuclear reactions, elucidate the terms:

(i) exoergic,

(ii) endoergic,

(iii) Q-value.

(b) Consider a nuclear reaction where an incident particle of mass M_i and kinetic energy T_i collides with a stationary target of mass M_t. After the reaction a particle is emitted with mass M_e and kinetic energy T_e leaving a residual nucleus with mass M_r and kinetic energy T_r. The emission takes place in a direction making an angle θ to the incident direction.

Obtain an expression of Q-value of this reaction. (1+1+1)+9=12

Group C

Answer *any six* questions, taking at least *one* from each section. 6×6=36

Section - I

10. Find the wavelength of radiation which will be emitted due to the transition of the electron in the singly ionized helium atom from the n = 4 to n = 2 level. You may neglect the mass of the electron compared to that of the nucleus in your calculation. 6

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11. Consider the hydrogen atom placed in an external, uniform, weak magnetic field $\vec{B}$ along the Z-axis. Taking into account the spin of the electron and the spin-orbit interaction, show that the magnetic energy of the atom, when its total angular momentum quantum number is j, is $g_j m_j B\mu_B$, where g_j is the Lande g-factor, m_j is the angular momentum projection quantum number and μ_B is the Bohr magneton. 6

Section - II

12. Two spaceships approach each other, each moving with the same speed as measured by a stationary observer on the Earth. Their relative speed is 0.70C. Determine the velocity of each spaceship as measured by the observer on the earth. 6

13. A target proton is at rest in the laboratory frame of reference. A projectile proton collides with this target proton and produces a π° meson by the reaction, $p + p \rightarrow p + p + \pi^\circ$.

Considering the two facts:

(i) for this reaction to be possible, the minimum kinetic energy available in the centre of mass frame (where the total momentum is always zero) is equal to the rest mass energy of π°,

(ii) the quantity $(E^2 - P^2 C^2)$ is independent of frame of reference,

Calculate the threshold kinetic energy (in MeV) of the projectile proton in the laboratory frame, for the π° production. 6

Section - III

14. Using the uncertainty principle, in the ground state (of size 'a') of hydrogen atom and making use of the following steps:

(i) Take size $a \approx \Delta x$,

(ii) Take $\Delta p_x \approx p$,

(iii) Write the total energy E of the electron,

determine the value of 'a' for which the energy is minimum. 6

15. (a) Write the one-dimensional time-independent Schrödinger equations with potential $V(x)$ for two energy eigenvalues E_n, E_k and corresponding eigenfunctions ψ_n, ψ_k.

(b) Using these equations show that

(i) the energy eigenvalues are real,

(ii) different energy eigenfunctions are orthogonal. 2+4=6

16. An otherwise free particle is constrained to move on the surface of a sphere.

(a) Write the Hamiltonian of this system in terms of the orbital angular momentum operator.

(b) Write the energy eigenfunctions and eigenvalues.

(c) Consider any one of the energy level and write the degree of degeneracy associated with it. Give reason for your answer. 2+(1+1)+(1+1)=6

1(h) The probability of finding the electron between r and $r+dr$ is

$$P(r)dr = |\psi_0|^2 r^2 dr \longrightarrow (1/2)$$

$$\therefore P(r) = |\psi_0|^2 r^2 = |A|^2 e^{-2r/a} r^2$$

$$\frac{dP(r)}{dr} = |A|^2 2r e^{-2r/a} + |A|^2 \left(-\frac{2}{a}\right) e^{-2r/a} r^2 \longrightarrow (1/2)$$

For maximum value of $P(r)$, $\frac{dP(r)}{dr} = 0 \longrightarrow (1/2)$

$$\Rightarrow 2r = \frac{2}{a} r^2$$

or, $r = a \longrightarrow (1/2)$

Q 1(i) The centrifugal potential $\frac{\hbar^2 l(l+1)}{2mr^2}$ has a sign which is opposite to that of the attractive coulomb potential. $\longrightarrow (1/2)$

Thus, it has an antibinding effect. $\longrightarrow (1/2)$

Hence, for $l=0$ the binding will be maximum and the s-state radial wavefunction will have maximum value at $r=0$. $\longrightarrow (1)$

Q 1(j) The range of nuclear force within a nucleus is quite small. As a result a nucleon can interact with its nearest neighbours only. $\longrightarrow (1/2)$

On the other hand, the repulsive Coulomb force betn the positively-charged protons is of long range. $\longrightarrow (1/2)$

There is a limit to the values of Z and N, beyond which the strong force cannot prevent the disruption of the nucleus by the repulsive effect. This limit is, $Z=83$, $A=209$. $\longrightarrow (1/2)$

Thus for $Z>83$, $A>209$, the heavy nuclei spontaneously transform into lighter nuclei by the emission of α-particles. $\longrightarrow (1/2)$

Q 1(k) As per the shell model,

(i) the first three shells $1s_{1/2}$, $2p_{3/2}$ and $2p_{1/2}$ can accommodate 8 protons and 8 neutrons following the Pauli principle; $\longrightarrow (1/2)$

(ii) The energy gap between the $2p_{1/2}$ shell and the next shell $3d_{5/2}$ is quite large so that the probability of excita[tion] from the former to the latter is very small; $\longrightarrow (1/2)$

(iii) this makes the ^{16}O nucleus ($N=8$, $Z=8$) a stable nucl[eus] having zero quadrupole moment, with spherically symm[etric] charge distribution

Q 1(l). In the process of β-decay of a nucleus, the electron is always associated with an antineutrino. → (½)

The maximum available energy which is the energy equivalent of the mass-difference between the parent and the daughter nuclei is continuously shared by the electron and the neutrino. → (1)

Therefore the electron energy spectrum has a continuous distribution. → (½)

Q 1(m) (i) The energy required to convert a liquid into vapour is directly proportional to the mass of the liquid. Thus the total binding energy of all the molecules in the liquid is directly proportional to the mass of the liquid; or the binding energy per unit mass is constant. This situation is similar to that in a nucleus, namely, the binding energy per nucleon is constant. → (1)

(ii) The density ~~in~~ an incompressible liquid is constant and does not depend on the shape or size of the liquid drop. This property is similar to that of a nucleus, namely, the density of nuclear matter is independent of the size of the nucleus. → (1)

Q 1(n) pi meson → spin quantum number = 0 → (½)

parity quantum number = −1 → (½)

rho meson → spin quantum number = 1 → (½)

parity quantum number = −1 → (½)

$\boxed{\text{GROUP} - \text{B}}$

Section - I

Q 2(a) Given in any standard book.

Marks distribution:

Photon energy-momentum before and after scattering → (1)

- Electron energy-momentum before and after scattering → (1)

Momentum conservation equations → (2)

Energy conservation equation → (1)

2(b) $\lambda'(\phi = 120^\circ) = \lambda + \frac{3h}{2mc}$ ⟶ (1/2)

$\lambda'(\phi = 60^\circ) = \lambda + \frac{h}{2mc}$ ⟶ (1/2)

$\lambda'(\phi = 120^\circ) = 2\lambda'(\phi = 60^\circ) \Rightarrow \lambda = \frac{h}{2mc}$ ⟶ (1/2)

$\therefore \lambda = \frac{6.626 \times 10^{-34}}{2 \times 9.109 \times 10^{-31} \times 2.998 \times 10^{8}}\ m$

$= 1.2 \times 10^{-3}\ nm$ ⟶ (1½)

Q2(c) Compton wavelength $\lambda_c = \frac{h}{mc}$

$= 2\lambda$

$= 2.4 \times 10^{-3}\ nm$ ⟶ (1)

Q3(a) The purpose of the Stern-Gerlach experiment was to know about the magnetic moment of electron associate with its spin. ⟶ (1)

If the magnetic field is uniform, then there is no net force on the magnetic moment. On the other hand, if the magnetic field is non-uniform, there is a net force on the magnetic moment. Thus, if the electron placed in a non-uniform magnetic field experiences a force, that indicates the presence of the magnetic moment. ⟶ (1)

Q3(b) Given in any standard book

Marks distribution:

Magnetic poles with correct shaps to produce non-uniform magnetic field ⟶ (2)

Beam of electrons from source ⟶ (1)

Photographic plate in its correct position ⟶ (1)

Observation on the photographic plate ⟶ (1)

3(c) When a beam of neutral atoms (such as of hydrogen or silver), is passed through a region of non-uniform magnetic field and is recorded on a photographic plate, it is seen to split into two parts. This observation signifies that z-component of spin has two values, which again demonstrates the fact that the spin quantum number of electron is ½ and corroborates the concept of space quantization. ⟶ (3)

The observation as per the expectation in classical physics should be a smearing of beam as recorde[d] by the photographic plate. $\longrightarrow$ 1

This is because, in classical physics, there is no space quantization and S_z has all possible values between $+\frac{1}{2}$ and $-\frac{1}{2}$ and not these two values only. $\longrightarrow$ 1

Q.4 (a) The motivation behind the Michelson-Morley experime[nt] was to verify whether an all-pervading medium or an absolute frame of reference, called 'ether' does exist, in which the speed of light is 'c', the value predicted from electromagneti[c] theory. $\longrightarrow$ 2

Q.4 (b) Given in Resnick's book.

Distribution of marks:

Simple diagram of Michelson interferometer showing beams 1 and 2 $\longrightarrow$ (

Diagram showing the cross-stream path of beam 2 $\longrightarrow$ (

Derivation of time t_1 taken by beam 1 $\longrightarrow$ (

Derivation of time t_2 taken by beam 2 $\longrightarrow$ 1

Derivation of time t_1' after 90° rotation of the instrument $\longrightarrow$ 1

Derivation of time t_2' after 90° rotation of the instrument $\longrightarrow$ (

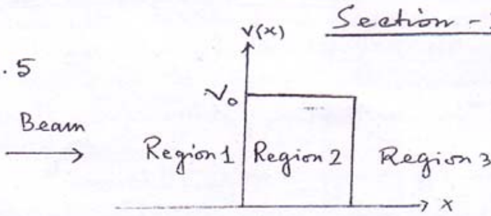
Derivation of the expression of ΔN $\longrightarrow$ 2

Q.4 (c) Experimentally observed value was, $\Delta N = 0$. (upper lim[it]) $\longrightarrow$ (

As the observed fringe shift was much below the expected value ($\Delta N = 0.4$), the implication of the observation was that there is no fringe shift at all and thus, ether does not exist. $\longrightarrow$ 1

Section - II

Q.5



Schrodinger equations:

Region 1: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$

$\Rightarrow \frac{d^2\psi}{dx^2} + k^2\psi = 0$, where $k^2 = \frac{2mE}{\hbar^2}$

Region 2: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi$

$\Rightarrow \frac{d^2\psi}{dx^2} - \alpha^2\psi = 0$ where $\alpha^2 = \frac{2m(V_0-}{\hbar^2}$

Region 3: $\frac{d^2\psi}{dx^2} + k^2\psi = 0$

The solutions of these equations are,

egion 1 : $\psi_1(x) = A e^{ikx} + B e^{-ikx}$

Region 2 : $\psi_2(x) = C e^{\alpha x} + D e^{-\alpha x}$ $\longrightarrow$ 1

Region 3 : $\psi_3(x) = F e^{ikx} + G e^{-ikx}$

Assume, no particles are incident from right. Hence $G=0$.

Matching ψ_1 and ψ_2 and their derivatives at $x=0$, we ge[t]

$$\psi_1(x)\Big|_{x=0} = \psi_2(x)\Big|_{x=0} \Rightarrow A+B = C+D \qquad (1)$$

$\longrightarrow$ 1

$$\frac{d\psi_1(x)}{dx}\Big|_{x=0} = \frac{d\psi_2(x)}{dx}\Big|_{x=0.} \Rightarrow ik(A-B) = \alpha(C-D) \qquad (2)$$

Matching ψ_2 and ψ_3 and their derivatives at $x=a$, we get,

$$\psi_2(x)\Big|_{x=a} = \psi_3(x)\Big|_{x=a} \Rightarrow C e^{\alpha a} + D e^{-\alpha a} = F e^{ika} \qquad (3)$$

$\longrightarrow$ 1

$$\frac{d\psi_2(x)}{dx}\Big|_{x=a} = \frac{d\psi_3(x)}{dx}\Big|_{x=a} \Rightarrow \alpha\left(C e^{\alpha a} - D e^{-\alpha a}\right) = ikF e^{ika} \qquad (4)$$

From (1) and (2), we get,

$$C = \frac{1}{2}\left(1+\frac{ik}{\alpha}\right)A + \frac{1}{2}\left(1-\frac{ik}{\alpha}\right)B \qquad (5)$$

$$D = \frac{1}{2}\left(1-\frac{ik}{\alpha}\right)A + \frac{1}{2}\left(1+\frac{ik}{\alpha}\right)B \qquad (6)$$

From (3) and (4) we get,

$$C = \frac{1}{2}\left(1+\frac{ik}{\alpha}\right)F e^{-\alpha a} e^{ika} \qquad (7)$$

$$D = \frac{1}{2}\left(1-\frac{ik}{\alpha}\right)F e^{\alpha a} e^{ika} \qquad (8)$$

$\longrightarrow$ 1

From (5), (6), (7) and (8), we get,

$$\frac{C}{D} = \frac{\left(1+\frac{ik}{\alpha}\right)}{\left(1-\frac{ik}{\alpha}\right)} e^{-2\alpha a} = \frac{\left(1+\frac{ik}{\alpha}\right)A + \left(1-\frac{ik}{\alpha}\right)B}{\left(1-\frac{ik}{\alpha}\right)A + \left(1+\frac{ik}{\alpha}\right)B}$$

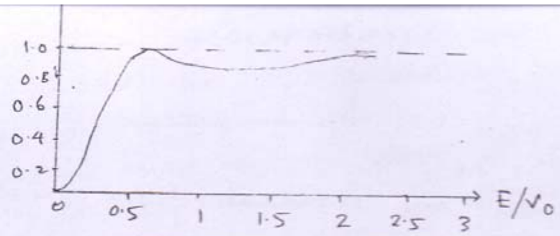
$$\Rightarrow \frac{B}{A} = \frac{(k^2+\alpha^2)(1-e^{-2\alpha a})}{(\alpha+ik)^2 e^{-2\alpha a} - (\alpha-ik)^2} = \frac{(k^2+\alpha^2)\sinh\alpha a}{(k^2-\alpha^2)\sinh\alpha a + 2i\alpha k\cosh\alpha a} \longrightarrow 1$$

$$R = \frac{|B|^2}{|A|^2} = \frac{(k^2+\alpha^2)^2\sinh^2\alpha a}{(k^2-\alpha^2)^2\sinh^2\alpha a + 4\alpha^2k^2\cosh^2\alpha a} \quad \text{(Reflection coefficient)} \rightarrow$$

$$T = 1-R = \frac{4\alpha^2k^2}{(k^2-\alpha^2)\sinh^2\alpha a + 4\alpha^2k^2\cosh^2\alpha a} \quad \text{(Transmission coefficient}$$

$\longrightarrow$ 2

[Note: Since there is no specification regarding the final forms of R and ... full credit.]



$\longrightarrow$ (2)

Q.6 (a) Given in any standard book $\longrightarrow$ (2)

(b) Given in any standard book

If $\psi(r, \theta, \phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$, then

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \left[\frac{2m}{\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0 r} + E\right) - \frac{l(l+1)}{r^2}\right]R = 0 \longrightarrow \text{(Derivation)} \longrightarrow ($$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\,\frac{d\Theta}{d\theta}\right) + \left[l(l+1) - \frac{m_l^2}{\sin^2\theta}\right]\Theta = 0 \longrightarrow \text{(Derivation)} \longrightarrow (3)$$

$$\frac{d^2\Phi}{d\phi^2} + m_l^2\,\Phi = 0 \longrightarrow \text{(Derivation)} \longrightarrow (2)$$

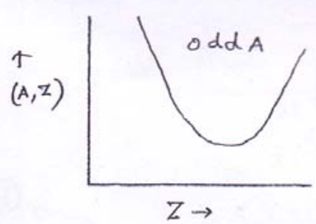
(c) $n = 3$, $l = 0, 1, 2$, $\longrightarrow$ (1)

$m_l = 0, \pm 1, \pm 2$ $\longrightarrow$ (1)

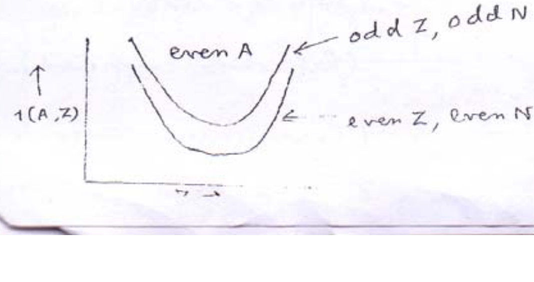
Section - III

2.7 Energy terms are elucidated in any standard book, e.g., Nuclear Physics by S. N. Ghoshal, pages 373-375.

Marks distribution: volume energy $\longrightarrow$ (1)
surface energy $\longrightarrow$ (2)
coulomb energy $\longrightarrow$ (2)
asymmetry energy $\longrightarrow$ (2)
pairing energy $\longrightarrow$ (2)



For odd-A nuclei, the pairing energy term $\delta = 0$. Hence, there is a single parabola. $\longrightarrow$ Graph (1/2) + Explanation (1/2)



For odd Z, odd N nuclei, the pairing energy term is positive.

For even Z, even N nuclei, the pairing energy term is negative. Hence there are two parabolas for even A nuclei. $\longrightarrow$ Graph (1) + Explanat (1)

.8 (a) See 'Nuclear Physics' by S.N. Ghoshal,
page 248 for diagram $\longrightarrow$ (2)
page 249 for description of parts $\longrightarrow$ (2)

(b) See Ghoshal's book, pages 250-251
Graph $\longrightarrow$ (1)
Explanation of shape $\longrightarrow$ (3)

(c) Quenching is necessary to prevent spurious counts. $\longrightarrow$ (1)

See Ghoshal's book, page 251, for the quenching process in a self-quenching counter $\longrightarrow$ (3)

2.9 (a) (i) Reactions during which energy is evolved are called exoergic reactions. $\longrightarrow$ (1)

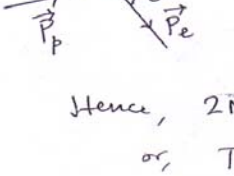
(ii) Reactions during which energy is absorbed are called endoergic reactions. $\longrightarrow$ (1)

(iii) The total amount of energy evolved or absorbed during a nuclear reaction is called the Q value of the reaction. $\longrightarrow$ (1)

(b) Definition of Q-value, $Q = (M_i + M_t)c^2 - (M_e + M_r)c^2 \longrightarrow$ (1)

Applying conservation of total energy which is equal to (rest mass energy + kinetic energy) we can write,

$$Q = T_e + T_r - T_i - T_t = T_e + T_r - T_i \quad (\because T_t = 0) \longrightarrow 1+1 = (2)$$



From conservation of momentum,

$$\vec{P}_i = \vec{P}_r + \vec{P}_e \longrightarrow (1)$$

$$\text{or } P_r^2 = P_i^2 + P_e^2 - 2|\vec{P}_i||\vec{P}_e|\cos\theta \longrightarrow (1)$$

Nonrelativistically, $T = \frac{P^2}{2M}$ or $P^2 = 2MT \longrightarrow$ (1)

Hence, $2M_rT_r = 2M_iT_i + 2M_eT_e - 2\sqrt{2M_iT_p}\cdot\sqrt{2M_eT_e}\cos\theta$

or, $T_r = \frac{M_i}{M_r}T_i + \frac{M_e}{M_r}T_e - \frac{2\sqrt{M_iM_eT_iT_e}}{M_r}\cos\theta$

$$Q = T_e + T_r - T_i = \left(\frac{M_i - M_r}{M_r}\right)T_i + \left(\frac{M_e + M_r}{M_r}\right)T_e - 2\frac{\sqrt{M_iM_eT_iT_e}}{M_r}\cos\theta \longrightarrow ($$

Section - I

Q.10 From Bohr's theory,

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0^2h^2}\left(\frac{1}{n^2}\right) \longrightarrow (1)$$

For helium, $Z^2 = 2^2 = 4$, $\therefore E_n = -\frac{me^4}{2\epsilon_0^2h^2}\left(\frac{1}{n^2}\right) \longrightarrow (1/2)$

Frequency of emitted radiation for transition from $n = 4$ to $n = 2$ levels is

$$\nu = \frac{E_{n=4} - E_{n=2}}{h} = -\frac{me^4}{2\epsilon_0^2h^3}\left(\frac{1}{16} - \frac{1}{4}\right) = \frac{3me^4}{32\epsilon_0^2h^3} \longrightarrow (1)$$

Wavelength of emitted radiation,

$$\lambda = \frac{c}{\nu} = \frac{32\epsilon_0^2h^3c}{3me^4} \longrightarrow (1/2)$$

$$= \frac{32 \times 8.854^2 \times 6.626^3 \times 2.998 \times 10^{-24} \times 10^{-102} \times 10^{8}}{3 \times 9.109 \times 10^{-31} \times 1.602^4 \times 10^{-\cancel{72}\,76}}\ m$$

$$= 12155 \cancel{\times 10^{-15}\,m} \times 10^{-11}\ m = 121.5\ nm$$

$$\cancel{= 0.12 \times 10^{-10}\,m} \longrightarrow (3)$$

[Note :- No extra credit will be given for the derivation of expression of E_n from Bohr's theory]

Q.11 The magnetic moments due to the orbital angular momentum $\vec{L}$ and spin angular momentum $\vec{S}$ are

$$\vec{\mu}_L = -\frac{e}{2m}g_\ell\vec{L}, \quad \vec{\mu}_S = -\frac{e}{2m}g_s\vec{S}, \text{ where } g_\ell = 1,\ g_s = 2$$

The total magnetic moment $\vec{\mu} = \vec{\mu}_L + \vec{\mu}_S = -\frac{e}{2m}(g_L\vec{L} + g_S\vec{S}) \longrightarrow (1)$

Define a vector $\vec{\mu}_J$ with its direction along $\vec{J}$ and magnitude given by the projection of $\vec{\mu}$ along $\vec{J}$. Thus,

$$\vec{\mu}_J = |\vec{\mu}_J|\frac{\vec{J}}{|\vec{J}|}, \quad |\vec{\mu}_J| = \frac{\vec{\mu}\cdot\vec{J}}{|\vec{J}|}$$

combining these two, we write,

$$\vec{\mu}_J = \frac{(\vec{\mu}\cdot\vec{J})\,\vec{J}}{|\vec{J}|^2}$$

Now, $\vec{\mu}\cdot\vec{J} = -\frac{e}{2m}(g_\ell\vec{L} + g_s\vec{S})\cdot\vec{J}$; hence, $\vec{\mu}_J = -\frac{e}{2m}\frac{(g_\ell\vec{L} + g_s\vec{S})\cdot\vec{J}}{|\vec{J}|^2}\vec{J}$

$$= -\frac{e}{2m}g_j\vec{J} \longrightarrow (1)$$

where $g_j = \frac{(g_\ell\vec{L} + g_s\vec{S})\cdot\vec{J}}{|\vec{J}|^2}$

Now, $\vec{L}\cdot\vec{J} = \vec{L}\cdot(\vec{L} + \vec{S}) = \vec{L}^2 + \vec{L}\cdot\vec{S} = \vec{L}^2 + \frac{\vec{J}^2 - \vec{L}^2 - \vec{S}^2}{2}$

$$\left.\begin{array}{l} \therefore (\vec{L}\cdot\vec{J})_{\ell,s,j} = \hbar^2\left[\ell(\ell+1) + \frac{j(j+1) - \ell(\ell+1) - s(s+1)}{2}\right] \\ \text{[illegible]}\ (\vec{S}\cdot\vec{J})_{\ell,s,j} = \hbar^2\left[s(s+1) + \frac{j(j+1) - \ell(\ell+1) - s(s+1)}{2}\right] \end{array}\right\} \longrightarrow (1)$$

...en we get,

$$g_j = \frac{1}{j(j+1)}\left[g_\ell \ell(\ell+1) + g_s s(s+1) + (g_\ell + g_s)\frac{j(j+1) - \ell(\ell+1) - s(s+1)}{2}\right].$$

Putting $g_\ell = 1$, $g_s = 2$, we get,

$$g_j = \frac{1}{2j(j+1)}\left[3j(j+1) + s(s+1) - \ell(\ell+1)\right], \longrightarrow (1)$$

which is the Lande g-factor.

The magnetic energy of the hydrogen atom is

$$E_M = -\vec{\mu}_j \cdot \vec{B} = \frac{e}{2m} g_j \vec{J}\cdot\vec{B} = \frac{eB}{2m} g_j J_z, \longrightarrow (1)$$

since the magnetic field is in the z-direction

Hence $E_M = \frac{e\hbar}{2m} g_j m_j B = g_j m_j B \mu_B$,

$\mu_B = \frac{e\hbar}{2m}$, being the Bohr magneton. $\longrightarrow (1)$

Section - II

.. 12.

We have the Lorentz velocity transformation equation,

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2}$$

where u_x is the velocity of an object measured in the O-frame, u_x' is the velocity measured in the O'-frame and v is is the velocity of O'-frame along the x-axis of O-frame. $\longrightarrow \frac{1}{2}+\frac{1}{2}=($

We take the O-frame attached to the earth and O'-frame to be attached to the spaceship moving to the right with velocity v. $\longrightarrow (1)$

The other spaceship has velocity $u_x = -v$ in O and has velocity $u_x' = -0.70$ in O'. $\longrightarrow (1)$

Then, we have, $-0.70c = \frac{-v - v}{1 + v^2/c^2}$

or, $0.70 = \frac{2v/c}{1+v^2/c^2} = \frac{2\beta}{1+\beta^2}$

or, $0.70\beta^2 - 2\beta + 0.70 = 0$ $\longrightarrow (1)$

$$\beta = \frac{2 \pm \sqrt{4 - 4\times0.70\times0.70}}{2\times0.70}$$

$= 2.45,\ 0.41$ $\longrightarrow (1)$

$\beta = 2.45$ is not acceptable. Hence, $\frac{v}{c} = 0.41$

or, $v = 0.41c$ $\longrightarrow (1)$

Q. 13

$$E_{cm}^2 = E_{lab}^2 - P_{lab}^2 c^2 \longrightarrow (1)$$

$$E_{cm} = (938 + 938 + 135)\,\text{MeV} = 2011\ \text{MeV} \longrightarrow (1)$$

Let the threshold K.E. in the lab frame be T

$$\therefore E_{lab} = 938 + 938 + T = (1876 + T)\ \text{MeV} \longrightarrow (1)$$

$$P_{lab}^2 c^2 = (938+T)^2 - 938^2$$

$$\therefore 2011^2 = (1876+T)^2 - (938+T)^2 + 938^2 \longrightarrow (1)$$

$$= 1876^2 + 2\times 1876 \times T + \cancel{T^2} - \cancel{938^2} - \cancel{T^2} - 2\times 938 \times T + \cancel{938^2}$$

$$= 1876^2 + 2\times 938 \times T$$

$$\text{or,}\quad T = \frac{(2011+1876)\times(2011-1876)}{2\times 938}$$

$$= 279.7\ \text{MeV.} \longrightarrow (2)$$

Section III

Q. 14

Uncertainty principle: $\Delta x \Delta p_x \approx \hbar \longrightarrow (1)$

Here $\Delta x \approx a$, $\Delta p_x \approx p$ $\therefore p \approx \hbar/a$

$$K.E. = \frac{p^2}{2m} \approx \frac{\hbar^2}{2ma^2}$$

$$P.E. = -\frac{q^2}{4\pi\epsilon_0 a}$$

$$\therefore E = K.E. + P.E.$$

$$= \frac{\hbar^2}{2ma^2} - \frac{q^2}{4\pi\epsilon_0 a} \longrightarrow (1)$$

For ground state $\frac{dE}{da} = 0$

$$\text{or,}\quad -\frac{\hbar^2}{ma^3} + \frac{q^2}{4\pi\epsilon_0 a^2} = 0$$

$$\Rightarrow a = \frac{\hbar^2}{m(q^2/4\pi\epsilon_0)} \longrightarrow (2)$$

$$= \frac{h^2}{4\pi^2} \times \frac{4\pi\epsilon_0}{mq^2}$$

$$= \frac{\epsilon_0 h^2}{\pi m q^2} = \frac{8.854\times10^{-12} \times 6.626^2 \times 10^{-68}}{3.14 \times 9.109\times10^{-31} \times 1.602^2 \times 10^{-38}}\ m$$

$$= 0.53 \times 10^{-10}\ m \longrightarrow (2)$$

$$\left.\begin{aligned} -\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} + V(x)\psi_n &= E_n\psi_n \\ -\frac{\hbar^2}{2m}\frac{d^2\psi_k}{dx^2} + V(x)\psi_k &= E_k\psi_k \end{aligned}\right\} \longrightarrow 2$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\left[\psi_k^*\frac{d^2\psi_n}{dx^2} - \psi_n\frac{d^2\psi_k^*}{dx^2}\right] = (E_n - E_k^*)\psi_k^*\psi_n$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\int_{-\infty}^{+\infty}\frac{d}{dx}\left[\psi_k^*\frac{d\psi_n}{dx} - \psi_n\frac{d\psi_k^*}{dx}\right]dx = (E_n - E_k^*)\int_{-\infty}^{+\infty}\psi_k^*\psi_n\,dx$$

The L.H.S. $= -\frac{\hbar^2}{2m}\left[\psi_k^*\frac{d\psi_n}{dx} - \psi_n\frac{d\psi_k^*}{dx}\right]\Bigg|_{-\infty}^{+\infty} = 0$, since the wave functions vanish at infinity

$$\Rightarrow \quad (E_n - E_k^*)\int_{-\infty}^{+\infty}\psi_k^*\psi_n\,dx = 0 \longrightarrow 2$$

If $n = k$, since $\int_{-\infty}^{\infty}|\psi_k|^2 dx$ is nonzero, $E_k^* = E_k$, showing that the energy eigenvalues are real $\longrightarrow$ 1

If $n \neq k$, then $\int_{-\infty}^{+\infty}\psi_k^*\psi_n\,dx = 0$, showing that the different energy eigenfunctions are orthogonal $\longrightarrow$ 1

1.16

(a) The Hamiltonian, $\hat{H} = \frac{\hat{L}^2}{2I}$ $\longrightarrow$ 2

where $\hat{L}$ is the orbital angular momentum operator and I is the moment of inertia.

(b) The energy eigenfunctions are the spherical harmonics $Y_{\ell m}(\theta, \varphi)$.. $\longrightarrow$ 1

The corresponding energy eigenvalues are

$E_\ell = \frac{\hbar^2}{2I}\ell(\ell+1)$, $\ell = 0, 1, 2, 3$ $\longrightarrow$ 1

(c) Every energy level E_ℓ is $(2\ell+1)$-fold degenerate. $\longrightarrow$ 1

This is because $\hat{H}$ commutes with $\hat{L}$ and thus, $\hat{H}$ is independent of the orientation in space; hence the energy spectrum does not depend on the component of $\hat{L}$ in any particular direction. $\longrightarrow$ 1

Section - IV

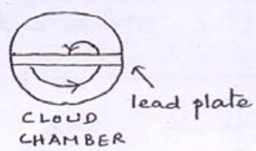
Q. 17 (a) The nucleus, after the emission of α and β-particles, may be left in an excited state. It then makes transition to states of lower energy by the emission of γ-rays. $\longrightarrow$ (1)

The range of energies of these γ-rays is from a few thousand electron volts to several million electron volts. $\longrightarrow$ (1)

(b) The bound electrons are emitted as photoelectron $\longrightarrow$ (1)
A bound electron can absorb the entire energy of the photon. On the other hand a free electron ~~can not~~ cannot absorb the entire energy of a photon, since the energy and momentum conservation can not be satisfied simultaneously in this case. $\longrightarrow$ (1)

(c) The minimum energy is 1.02 MeV. $\longrightarrow$ (1)
The pair-creation is not possible in vacuum because in that case the energy and momentum conservations cannot be simultaneously satisfied. $\longrightarrow$ (1)

Q. 18 (a)



$\longrightarrow$ (1)

Anderson set up a cloud chamber in a vertical plane, in a magnetic field of 15,000 gauss. In the centre of the cloud chamber was a horizontal lead plate 6 mm thick. Among some 13,000 exposures taken with this apparatus, several pictures appeared which suggested the presence of a light, positively charged particle. Ionizing tracks were observed with a relatively large radius of curvature in the lower half of the cloud chamber, apparently passing through the lead plate, and emerging on the upper side with considerably smaller radius of curvature. The sense of the magnetic field was such that negative particles curve to the left if travelling downward, to the right if travelling upward. The measured radius of curvature showed that they were not proton, but a positively charged particle which is of the same mass as that of electron. In this way, the positron was discovered.

(b) The constituents of cosmic ray showers are electron-positron pairs. → (1)
They are produced by the impact of high speed particles or photons upon matter. → (1)

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19 (a) electromagnetic → (1)
(b) weak → (1)
(c) weak → (1)
(d) strong → (1)
(e) electromagnetic → (1)
(f) weak → (1)