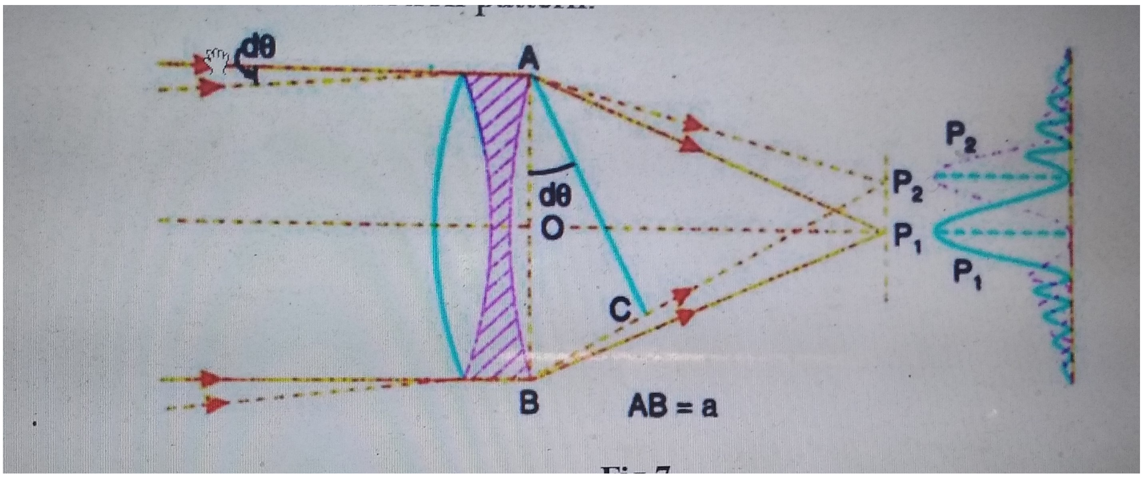
SEM-II
Hons (C IV: WAVES AND OPTICS)
L-5
Manoj Kumar Saha,
Assistant Professor, Department of Physics K C College Hetampur

Resolving power of a telescope

Let a be the diameter of the objective of the telescope given in $fig-1$. Consider the rays of light from two neighboring points of a distant object. The image of each point object is a Fraunhofer diffraction pattern.

Let $P1$ and$P2$ be the positions of the central maxima of the two images. According to Rayleigh, these two images are said to be resolved if the position of the central maximum of the second image coincides with the first minimum of the first image and vice versa. The path difference between the secondary waves traveling in the directions$AP1$ and$BP1$ is zero and hence they reinforce with one another at $P1$ Similarly, all the secondary waves from the corresponding points between A and B will have zero path difference. Thus$P1$ corresponds to the position of the central maximum of the first image.

The secondary waves traveling in the directions$AP2$ and $BP2$ will meet at $P2$ on the screen. Let the angle $P2AP1$ be $d\theta$. The path difference between the secondary waves traveling in the directions $BP2$ and $AP2$ is equal to BC.
From the triangle ABE , BC=AB



$$Sin\theta d\theta = AB.d\theta = a.\theta \tag{1}$$

If this path difference the position of$P2$ corresponds to the first minimum of the first image. But $P2$ also is the position of the central maximum of the second image. Thus, Rayleigh's condition of resolution is satisfied if
The whole aperture AB can be considered to be made of two halves AO and OB. The path difference between the secondary waves from the corresponding points in the two halves will be . All the secondary waves destructively interfere with oneanother and hence $P2$ will be the first minimum of the first image.The equation holds good for rectangular apertures. For circular apertures,this equation, according to Airy, can be written as,

where λ is the wavelength of light and a is the aperture of the telescope objective. Theaperture is equal to the diameter of the metal ring which the objective lens is mounted.Here refers to the limit of resolution of the telescope.

The reciprocal of $d\theta$ measures the resolving power of the telescope.

$$\frac{1}{d\theta} = \frac{a}{1.222\lambda} \tag{2}$$

$d\theta$ is also the angle subtended by the two distant object points at the objective. From equation (3), it is clear that a telescope with large diameter of the objective has higher resolving power,

Thus, resolving power of a telescope can be defined as the reciprocal of the angular separation that two distant object points must have, so that their images will appear just resolved according to Rayleigh's criterion.

I f f is the focal length of the telescope objective, then,

$$d\theta = \frac{r}{f} = \frac{1.22\lambda}{a} \tag{3}$$

$$r = \frac{1.22f\lambda}{a} \tag{4}$$

where r is the radius of the central bright image. The diameter of the first dark ring is equal to the diameter of the central image. The central bright disc is the Airy's disc. From equation (4), if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction patterns will appear sharper and the angular separation between the two just resolvable point objects will be smaller. Correspondingly, the resolving power of the telescope will be higher.

Example 1: Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is $3.8 \times 10^5 km$. The eye is most sensitive to light of wavelength $5500A$.

Solution: The limit of resolution of telescope is given by d$\theta = \frac{1.22\lambda}{a}$

Here $\lambda = 5500A^0 = 5500 \times 10^{-8}\ cm,\ \ a = 500cm$

$d\theta = \frac{1.22 \times 5500 \times 10^{-8}\ cm}{500\ cm}$

$d\theta = 13.42 \times 10^{-8}$

Let the distance between two points be x.

$d\theta = \frac{x}{R}$

Where $R = 3.8 \times 10^5 km$

So $x = R\ d\theta$

$x = 3.8 \times 10^{10}\ cm \times 13.42 \times 10^{-8}$

$x = 51.0m$