

SEM-IV
PHYH-C IX: ELEMENTS OF MODERN PHYSICS

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Solution of mathematical problems L-1, L-2, L-3

Mathematical prob. of L1, L2, L3, L4.

L-1.

prob. 1.

$$h = 6.627 \times 10^{-27} \text{ erg s}$$
$$e = 4.8 \times 10^{-19} \text{ esu}$$
$$m = 9.1 \times 10^{-28} \text{ g}$$

work function $W_0 = 2.28 \text{ eV} = 2.28 \times 1.6 \times 10^{-12} \text{ ergs}$

$$\therefore \frac{1}{2} m v_{\text{max}}^2 = h\nu - h\nu_0$$
$$= h\nu - W_0$$
$$\therefore v_{\text{max}}^2 = \frac{2h\nu - W_0}{m}$$

we know for u.v ray velocity = velocity of light = $3 \times 10^{10} \text{ cm/s}$.

$$c = \nu \lambda$$
$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^{10}}{3000 \times 10^{-8}} = 10^{15} \text{ Hz}$$
$$\therefore v_{\text{max}}^2 = \frac{2 \times (6.627 \times 10^{-27} \times 10^{15} - 2.28 \times 1.6 \times 10^{-12})}{9.1 \times 10^{-28}}$$
$$v_{\text{max}} = \sqrt{\frac{5.958}{9.1} \times 10^8}$$
$$= 8.1 \times 10^7 \text{ cm s}^{-1}$$
$$\underline{v_{\text{max}} = 8.1 \times 10^5 \text{ m s}^{-1}}$$

Ans/ly

L-1
Prob-2

From Einstein photoelectric equation

$$h\nu_1 = \frac{1}{2} m v_1^2 + W_0 \quad \dots \textcircled{1}$$

$$h\nu_2 = \frac{1}{2} m v_2^2 + W_0 \quad \dots \textcircled{2}$$

Subtracting ① from ② we get-

$$\begin{aligned} h(\nu_2 - \nu_1) &= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \\ &= (K.E)_2 - (K.E)_1 \end{aligned}$$

$$h = \frac{(K.E)_2 - (K.E)_1}{\nu_2 - \nu_1}$$

$$= \frac{(K.E)_2 - (K.E)_1}{\frac{c}{\lambda_2} - \frac{c}{\lambda_1}}$$

$$\therefore \lambda_2 = c.$$

$$= \frac{(4.0 - 1.8) \times 1.6 \times 10^{-19}}{3 \times 10^8 \left(\frac{1}{700} - \frac{1}{800} \right) \times 10^{10}}$$

$$= \underline{6.57 \times 10^{-34} \text{ J s}}$$

L-1
Prob-3.

We know from Einstein's photoelectric equation

$$h\nu = \frac{1}{2} m v_{\text{max}}^2 + W_0$$

$$\therefore W_0 = h\nu - \frac{1}{2} m v_{\text{max}}^2$$

$$= (6.62 \times 10^{-27} \times 7.5 \times 10^{14}) - (1.6 \times 10^{-19} \times 10^7)$$

$$= 3.365 \times 10^{-12} \text{ erg.}$$

$$\therefore W_0 = h\nu_0$$

$$\nu_0 = \frac{W_0}{h} = \frac{3.365 \times 10^{-12}}{6.62 \times 10^{-27}} = 5.08 \times 10^{14} \text{ Hz.}$$

L-3.

Prob. 4.

From Einstein's photoelectric equation

$$h\nu = h\nu_0 + \frac{1}{2} m v_{\max}^2$$

$$\therefore \frac{1}{2} m v_{\max}^2 = h\nu - h\nu_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$\therefore hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) = \frac{1}{2} m v_{\max}^2$$

$$\therefore \frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{1}{2} \cdot m \cdot \frac{v_{\max}^2}{hc}$$

$$\therefore \frac{1}{\lambda} - \frac{1}{\lambda_0} = \frac{1}{2} \cdot \frac{9.11 \times 10^{-31} \times (4 \times 10^5)^2}{(6.626 \times 10^{-34}) \times 3 \times 10^8}$$

$$= 0.0366 \times 10^7$$

$$\therefore \frac{1}{\lambda_0} = \frac{1}{\lambda} - 0.0366 \times 10^7$$

$$\therefore \frac{1}{\lambda_0} = \frac{1}{6000 \times 10^{-10}} - 0.0366 \times 10^7$$

$$= \frac{1}{6} \times 10^7 - 0.0366 \times 10^7$$

$$= 10^7 \left(\frac{1}{6} - 0.0366 \right)$$

$$= 0.13 \times 10^7$$

$$\therefore \lambda_0 = 7.692 \times 10^{-7} \text{ m}$$

$$= 7692 \times 10^{-10} \text{ m}$$

$$= \underline{\underline{7692 \text{ \AA}}}$$

L-2.

Prob-1. (a) $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0c} (1 - \cos\phi)$
 $= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{9.1 \times 10^{-31} \times 3 \times 10^8} (1 - \cos 95^\circ)$
 $= \underline{7.1 \times 10^{-13} \text{ m}}$

(b) $\lambda' = \lambda + \Delta\lambda$
 $= (2.426 \times 10^{-12} + 7.1 \times 10^{-13}) \text{ m}$
 $= \underline{3.1 \times 10^{-12} \text{ m}}$

(c) $\phi = \frac{\pi}{2}$ then $\lambda_c = \Delta\lambda = \frac{h}{m_0c} = 0.024 \text{ \AA}$

(d) Energy of the incident photon $E_i = \frac{hc}{\lambda} \Rightarrow$
 $= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.426 \times 10^{-12}}$
 $= 8.198 \times 10^{-14} \text{ J}$
 $= 0.51 \times 10^6 \text{ eV} \left[\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \right]$
 $= \underline{0.51 \text{ MeV}}$

(e) Energy of the scattered photon:

$$E_s = \frac{hc}{\lambda'}$$
$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3.1 \times 10^{-12}}$$
$$= 0.40 \text{ MeV}$$

(f) Energy loss by incident photon = $E_i - E_s$
 $= 0.51 \text{ MeV} - 0.40 \text{ MeV}$
 $= 0.11 \text{ MeV}$

(g) K.E of the recoil electron = energy loss by incident photon
 $E = E_i - E_s$
 $= 0.11 \text{ MeV}$

(h) The direction of emission of the corresponding recoil electron.

$$\tan \theta = \frac{\cot \frac{\phi}{2}}{1 + \frac{h\nu}{m_0 c^2}} \quad [\phi = 90^\circ]$$

$$= \frac{\cot 45^\circ}{1 + \frac{h\nu}{m_0 c^2}}$$

$$= \frac{1}{2}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{2}\right) = \cancel{20} \quad 26.56^\circ.$$

(i) The total energy of recoil electron

= K.E of recoil electron + rest mass energy of recoil electron

$$= E + m_0 c^2$$

$$= 0.11 \text{ MeV} + \frac{9.1 \times 10^{-31} \times (3 \times 10^8)^2}{1.6 \times 10^{-19} \times 10^6} \text{ MeV}$$

$$= 0.11 \text{ MeV} + 0.51 \text{ MeV}$$

$$= 0.62 \text{ MeV}$$

$$(j) \text{ The fraction of energy loss} = \frac{E_1 - E_2}{E_1}$$
$$= \frac{0.11 \text{ MeV}}{0.51 \text{ MeV}} = 0.21$$

1-3

(1) In between a photon 100 eV and an electron of 100 eV, which one has shorter wavelength?

A: For photon energy $E = h\nu$
 $= \frac{hc}{\lambda}$
 $\therefore \lambda = \frac{hc}{E}$
 $= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{100 \times 1.6 \times 10^{-19}} = 121 \text{ \AA}$

But for an electron like particle de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$
$$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$
$$= 4.2 \times 10^{-10} \text{ m}$$
$$= 1.2 \text{ \AA}$$

Hence de-Broglie wavelength of the electron is much smaller than that of a photon for their same energy.

(2) what is the de-Broglie wavelength of a thermal neutron at 400K?

we know K.E of a particle at equilibrium absolute temperature

$$E = \frac{3}{2} kT$$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m \cdot \frac{3}{2} kT}} = \frac{h}{\sqrt{3mkT}}$$

$$\therefore \lambda = \frac{6.62 \times 10^{-34}}{\sqrt{3 \times (1.675 \times 10^{-27}) \times 1.38 \times 10^{-23} \times 400}} \text{ m}$$
$$= 1.25 \times 10^{-10} \text{ m} = 1.25 \text{ \AA}$$

(3) If an electron has a wavelength of 1 \AA . Find the energy and momentum.

For a particle of mass m and energy E , the de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2} \text{ J}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2 \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 150 \text{ eV.}$$

Now momentum $p = \frac{h}{\lambda}$

$$= \frac{6.62 \times 10^{-34}}{10^{-10}} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$$

$$= 6.62 \times 10^{-24} \text{ kg} \cdot \text{m} \cdot \text{s}^{-1}$$